Computer Lab Project No. 6

Confidence Intervals with known standard deviation

Today we are going to experiment with confidence intervals for the population mean.

Info

In this lab, we will use simulation, as discussed in Computer Lab 4, the normal calculator, treated in Computer Lab 5, and methods you learned in class, for estimating the population mean when the *population standard deviation is known*. No new StatCrunch material this time!

Do now

1. Simulate rolling a fair die 100 times, using StatCrunch. So you can open statcrunch directly, without loading a data set from the textbook. You'll need 3 samples which should be put in columns 1 to 3. Here is how this can be done (see also Computer Lab Project No. 4):
   (a) Click Data→Simulate Data→Discrete Uniform.
   (b) In the popup window, put "100" in the text field labeled "Rows".
   (c) Put "3" in the text field labeled "Columns".
   (d) Put "1" as minimum and "6" as maximum.
   (e) You can leave the other settings as they are. Click the "Simulate" button at the bottom right of the window.
2. Consider the experiment of rolling a fair die. On a sheet of paper, calculate the mean and the standard deviation. That is, you use the formulas you learned in class before Exam 2. The result of rolling a die is given by a uniformly distributed random variable taking the values 1, ..., 6, each value occurring with probability 1/6.

   The random numbers you generated can be viewed as being samples taken from a population with that mean and standard deviation.
3. Use what you learned in class to calculate confidence intervals for the population mean with confidence level 0.95 based on these samples. Note that even though these samples are realizations of the same distribution (are taken from the same population), the confidence intervals are not the same. The confidence intervals are calculated from, and thus dependent on, the specific sample used. Recall the formula for the confidence interval for the mean at confidence level α:
   $$(\bar{x} - E, \bar{x} + E), \text{ where } E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$
   Recall further that $z_{\alpha/2}$ is the score such that the area under the standard normal curve and to the right of the score is $\alpha/2$. In other words, $z_{\alpha/2}$ is the $100 \cdot (1 - \frac{\alpha}{2})$-th percentile of the standard normal distribution. Also, α is 1−the confidence level, so in our case, $\alpha = 0.05$. You can use StatCrunch to calculate $z_{\alpha/2}$, using the normal calculator, as discussed in the previous lab. Write down the value of $z_{\alpha/2}$ and the three confidence intervals you found.
4. In how many of your samples was the population mean contained in the confidence interval you calculated?
5. Repeat these steps from the beginning, but this time, calculate confidence intervals at confidence level 0.99.

Finish

Follow the steps given in the Lab 6 instructions above, and create a pdf file called "Lab6-Firstname-Lastname.pdf" containing the answers to the questions asked in the lab:

- What are the mean (expected value) and standard deviation of rolling a fair die?
- For confidence level .95, what are α and $z_{\alpha/2}$?
- What are the 95% confidence intervals you found for your three simulations?
- How many of the simulated samples resulted in a confidence interval that contains the population mean (i.e., the expected value)?
- Answer the questions corresponding to the last three bullet points when the confidence level is 99%.

Submit your file via Blackboard as your Lab 6 assignment.