

Recent Trends in Nonlinear PDE

Titles and Abstracts

The Graduate Center, CUNY
365 Fifth Avenue
New York, NY 10016
Science Center, Room 4102

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9:00am till 4:00pm

Xavier Cabré, ICREA and Universitat Politecnica de Catalunya, Spain
Saddle-shaped solutions to the scalar Ginzburg-Landau equation

We establish the uniqueness of a saddle-shaped solution to the diffusion equation $-\Delta u = f(u)$ in all of \mathbb{R}^{2m} , where u is real valued and f is of bistable type, in every even dimension $2m \geq 2$. In addition, we prove its stability whenever $2m \geq 14$.

Saddle-shaped solutions are odd with respect to the Simons cone $\mathcal{C} = \{(x^1, x^2) \in \mathbb{R}^m \times \mathbb{R}^m : |x^1| = |x^2|\}$ and exist in all even dimensions. Their uniqueness was only known when $2m = 2$. On the other hand, they are known to be unstable in dimensions 2, 4, and 6. Their stability in dimensions 8, 10, and 12 remains an open question. In addition, since the Simons cone minimizes area when $2m \geq 8$, saddle-shaped solutions are expected to be global minimizers when $2m \geq 8$, or at least in higher dimensions. This is a property stronger than stability which is not yet established in any dimension.

Rupert Franck, Princeton University

Uniqueness and nondegeneracy of ground states for non-local equations in dimension one

We prove uniqueness of energy minimizing solutions Q for the nonlinear equation $(-\Delta)^s Q + Q - Q^{\alpha+1} = 0$ in $1D$, where $0 < s < 1$ and $0 < \alpha < \frac{4s}{1-2s}$ for $s < 1/2$ and $0 < \alpha < \infty$ for $s \geq 1/2$. Here $(-\Delta)^s$ is the fractional Laplacian. As a technical key result, we show that the associated linearized operator is nondegenerate, in the sense that its kernel is spanned by Q' . This solves an open problem posed by Kenig, Martel and Robbiano.

Changfeng Gui, University of Connecticut
Entire Solutions of the Allen-Cahn Equation

The Allen-Cahn equation is a popular PDE model for phase transition and phase separation. The level sets of solutions represent interfaces between two materials. A fine analysis of interfaces relies on solutions of Allen-Cahn equation in the entire spaces. In this talk, I will give a brief survey on results for local minimizers of the related Allen-Cahn energy functional, in particular in connection with the De Giorgi conjecture which relates the level set of monotone solutions to minimal surfaces. I will further discuss a classification scheme for finite morse index solutions of Allen-Cahn equation in the whole plane. The level sets of such solutions may represent intersecting interfaces. The energy of such solutions displays a quantization effect and relates to the number of interfaces, which is similar to the energy quantization of entire solutions of Ginzburg-Landau equation where the number of vortices is related. In particular, I will show that entire solutions with four ends must have even symmetry.

Fang-Hua Lin, Courant Institute
An optimal partition problem for the Dirichlet eigenvalues

In this talk, I shall describe some joint work with Luis Caffarelli concerning an optimal partition problem for the Dirichlet eigenvalues, and show how it is equivalent to the study of properties of harmonic mapping into a singular space. In particular, we prove the smoothness of domain walls as well as the regularity of (triple) junctions. The gradient flow of this partition problem will also be discussed.

Organizers:

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