Recent advances in 3D Euler and Navier-Stokes equations

Schedule and Abstracts

The Graduate Center, CUNY
365 Fifth Avenue
New York, NY 10016
Science Center, Room 4102

Thursday, October 27th, 2011
9:00am till 4:00pm

9:00–9:30:	*Coffee*
9:30–10:30:	Nader Massmoudi
10:45–11:45:	Thomas Y. Hou
11:45–1:00:	*Lunch*
1:00–2:00:	Gabriel Koch
2:15–3:15:	Alexis Vasseur

Thomas Y. Hou, Caltech

The interplay between computation and analysis in the study of 3D incompressible flows

Whether the 3D incompressible Navier-Stokes equations can develop a finite time singularity from smooth initial data with finite energy is one of the Seven Millennium Problems posted by the Clay Mathematical Institute. We review some recent theoretical and computational studies of the 3D Euler equations which show that there is a subtle dynamic depletion of nonlinear vortex stretching due to local geometric regularity of vortex filaments. Our study shows that convection could have a stabilizing effect for certain flow geometry. This is demonstrated through two reduced models of the 3D incompressible Navier-Stokes equations. Finally we present a new class of solutions to the 3D Euler and Navier-Stokes equations which could lead to a strong nonlinear alignment in the vortex stretching term and have the potential to develop a finite time singularity. However, the fluid dynamic instability of the fluid flow eventually kicks in and destroys such nonlinear alignment dynamically.

Gabriel Koch, Oxford University, UK
An Alternative Approach to Regularity for the Navier-Stokes Equations in Critical Spaces
We use the dispersive method of “critical elements” established by C. Kenig and F. Merle to give an alternative proof of a well-known Navier-Stokes regularity criterion due to L. Escauriaza, G. Seregin and V. Sverak, namely that 3-d solutions for which the spatial L^3-norm of the velocity remains bounded in time cannot develop a singularity. Their result came as the difficult ”endpoint” of a range of regularity criteria due to J. Serrin. The key tool in our proof is a decomposition into ”profiles” of bounded sequences in critical spaces (e.g., L^3). As a byproduct, we also generalize a recent result of W. Rusin and V. Sverak on ”minimal blow-up data” for Navier-Stokes. We will also discus generalizations of the Escauriaza-Seregin-Sverak criterion in critical Besov spaces. This is based on joint works with C. Kenig and with I. Gallagher and F. Planchon respectively.

Nader Masmoudi, Courant Institute
Inviscid limit of the free boundary Navier-Stokes system.

We will describe the limit from the free boundary Navier-Stokes system with Navier boundary condition to the free boundary Euler system.

Alexis Vasseur, Oxford university, UK
Drift diffusion equations with fractional diffusion and the Surface Quasi-Geostrophic equation

Motivated by the critical dissipative quasi-geostrophic equation, we prove that drift-diffusion equations with L^2 initial data and minimal assumptions on the drift are locally Holder continuous. As an application we show that solutions of the quasi-geostrophic equation with initial L^2 data and critical diffusion $(-\Delta)^{1/2}$, are locally smooth for any space dimension.