MTH229

More on Graphing with MATLAB

Project 3– Exercises

NAME: _____
SECTION: _____
INSTRUCTOR: _____

Exercise 1:
To illustrate, type in the following commands, which plot the function $f(x) = \sin(x^2)$ over the interval $[0, \pi]$.

```
>> x = linspace(0, pi);
>> y = sin(x.^2);
>> plot(x,y); grid
>> zoom on                          % just zoom works here too
```

Now clicking in the figure window will cause the graph to be redrawn over a smaller domain.

a. Click near the smallest **positive** x in this interval where f(x)=0. (The positive x-value where the graph first crosses the x axis). What is the length of the interval shown on the y axis?
(1) Answer: _____

b. Click twice more near this point of first intersection. Estimate (no more than two decimal points) from the zoomed in graph the numeric value of the x for which $f(x) = 0$.
(2) Answer: _____

c. Click the - magnifying glass and zoom out. Then the + magnifying glass to zoom in. This time on the second intersection point. Click near it 3 times. Estimate this value of x for which $f(x) = 0$.
(3) Answer: _____

Exercise 2:
Replot the ball problem until you can find a good estimate for the value of b so that $y(t) \geq 0$

on the interval $[0, b]$, and is negative for $t > b$. What is the value of b that you found?
(4) Answer: _____

Exercise 3:
Find the suggested values by plotting the graphs until you can figure out the answer.

a. Repeating the above example, find one period of the function

$$f(x) = 120\sin(120\pi x).$$

(This is a bit tricky! Keep plotting until you clearly get one period shown in the viewing window.)
(5) Answer: _____

b. When does $f(x) = 25 - x - \sin(x)$ cross the x-axis? (Guess an answer to within 1 decimal point.)
(6) Answer: _____

Exercise 4:
From its graph, estimate the minimum value of the function

$$f(x) = \frac{5}{\cos(x)} + \frac{8}{\sin(x)}$$

over the interval $(0, \pi/2)$.

Just plotting with x values given by

```
>> x = linspace(0,pi/2)
```

will cause the same problems as above. Make a plot, judiciously avoiding the vertical asymptotes, and then find the minimum value (y value) of the function on this interval.
(7) Answer: _____

Exercise 5:
Although it isn't an asymptote, this next problem has the same problem with the default y-axis scale being too big to accurately see other features of the graph.

Let

$$f(x) = x^5 - 225x^3 - x^2 + 225.$$

Our goal is to find the three real roots (zeroes) of this polynomial.

a. First plot the graph on the interval $(-17, 17)$. What are the values of the x intercepts that you can *clearly* see.
(8) Answer: _____

b. Is 0 the other root? From the graph over $-17, 17$ it seems plausible. Explain why you know that 0 is not the other real root for this polynomial.
(9) Answer:

c. Replot the function over an appropriate domain to find the third real root:
(10) Answer: _____

Exercise 6:
Consider the functions of the previous example, $y_1 = 4\cos(x)$ and $y_2 = \cos(4x)$, make the graphs and use them to answer the following:

a. Which function is oscillating more rapidly?
(11) Circle one: **1.** $\cos(4x)$ **2.** $4\cos(x)$

b. Which function has the larger amplitude?
(12) Circle one: **1.** $\cos(4x)$ **2.** $4\cos(x)$

Exercise 7:
Let $f(x) = \sqrt{x}$. Plot $f(x)$ and $g(x) = f(x) - 5$ together over the interval $[0, 10]$. The graph of $g(x)$ is the graph of $f(x)$ shifted ...
(13) Circle one: **1. left** by 5 **2. down** by 5 **3. right** by 5 **4. up** by 5

Exercise 8:

We wish to graph a function and its tangent line. Graph both the function $y = \sqrt{x}$ over the interval $[0, 9]$ and the line through the point $(4, 2)$ with slope $1/4$.

(14) **Attach your graph to the worksheet.**

Exercise 9:
Plot a graph that describes the following calling plan: You pay \$35 for the first 1,000 minutes. After the first 1,000 minutes you pay \$0.30 per minute. Draw a graph for usage between 0 and 2,000 minutes.

a. What commands did you use?
(15)

b. (16) **Attach your graph to the worksheet.**

Exercise 10:
Suppose your salary was \$42,200 in 2003 and \$47,500 in 2005. You are interested in your salary in the year 2011, but of course you would like to know this **now**. What do you do? Well, you can try a mathematical model. There are two common ones:

Linear model This model says your salary will go up by a fixed amount each year (the slope). The formula is of the form

$$s = s_{2003} + m(t - 2003),$$

where t is the year and s_{2003} is the amount you make in the year 2003 which is \$42,200.

Exponential model This model assumes your salary increases by a fixed **percentage** each year (not *amount* as in the linear model). The general formula is of the form

$$s = s_{2003}e^{r(t-2003)}$$

where r is figured out from the data. We'll help you out. For this problem r is

$$r = \frac{\ln(47500/42200)}{2} = 0.0592...$$

Assume for now that your salary follows a "linear growth" pattern.

a. What MATLAB command calculates the salary values, s as a function of t?
(17) Answer:

b. Assuming linear growth continues through the year 2011, make a graph to display your salary from year 2003 to 2011. After the plot command, label the axes so that the x axis says "year", the y says "yearly salary," and the title says "My salary."

c. Use the graph to predict your salary in 2011.
(18) Answer: _____

Exercise 11:

a. On your graph, label the exponential model with "exponential" and an arrow, and the linear model with "linear" using the icons in the figure window menubar.

b. Use the exponential model to estimate your salary in the year 2011.
(19) Answer: _____

c. The difference between the linear model and exponential model is how they predict the the amount your raise will be each year. One predicts that your salary will increase by a fixed *amount* each year. The other that your salary will increase by a fixed **percent** each year.

Which model predicts that your raise will be a fixed percent each year?
(21) Circle one: **1.** linear model **2.** exponential model

d. What is the fixed percent each year that is predicted?
(22) Answer: _____

Exercise 12:
On the same graph, mark the two points $(2003, 42200)$ and $(2005, 47500)$ using diamonds (the `'d'` argument to `plot`).

Exercise 13:
On the same graph. Draw the line $s = 60,000$. Your graph should have 3 functions on it now: the linear model, the exponential model and the flat line $s = 60,000$.

a. In what year does the linear model predict that your salary will be at least \$60,000?
(23) Answer: _____

b. In what year does the exponential model predict that your salary will be at least \$60,000?
(24) Answer: _____

c. Attach your graph (all 3 functions and annotations).
(25) **Attach your graph to the worksheet.**

Exercise 14:
(Optional:for extra credit)

The radius r of the smallest circular paper plate that holds a pizza of a given radius R=5 inches and central angle θ, $0 < \theta \leq \pi$, is given by the function $r = f(\theta)$ where

$$\begin{aligned} r = \quad f(\theta) = \quad & \frac{R}{2\cos\frac{\theta}{2}} \quad \text{if } 0 < \theta \leq \tfrac{\pi}{2} \\ r = \quad f(\theta) = \quad & R\sin\tfrac{\theta}{2} \quad \text{if } \tfrac{\pi}{2} < \theta \leq \pi. \end{aligned}$$

Following sketches to be done by hand:

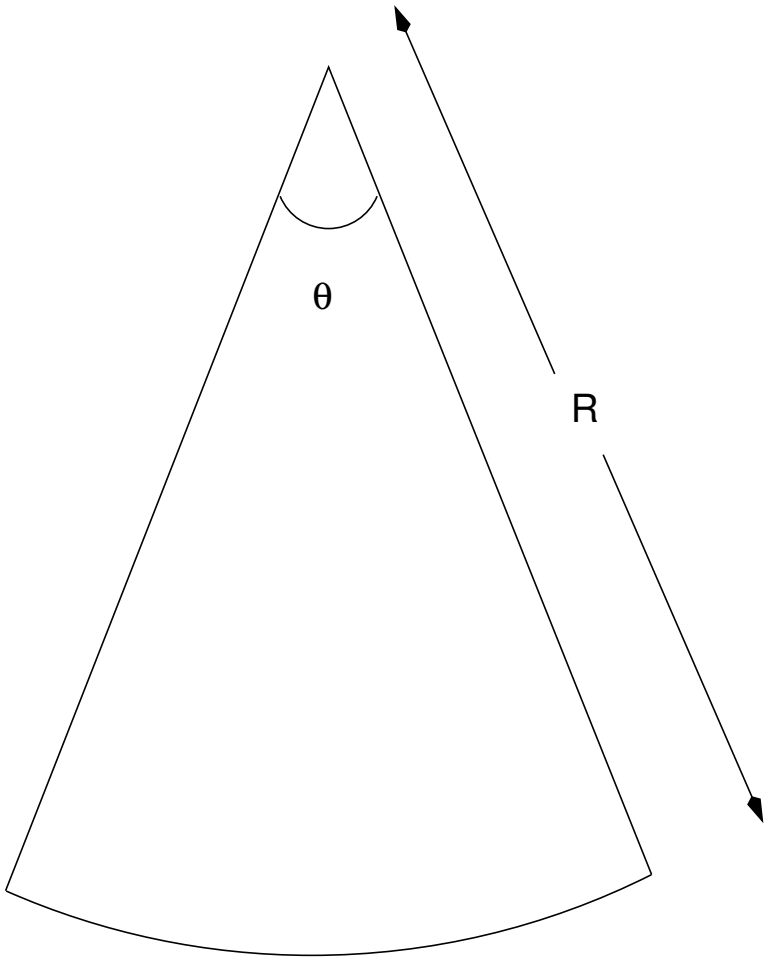
a. Sketch the pizza in its plate for $\theta = \pi/6$ (a 30 degree wedge). Find r for $\theta = \pi/6$.

b. Sketch the pizza in its plate for $\theta = \pi$ (half the pizza). Find r for $\theta = \pi$.

c. Determine the function $f(\theta)$ for the domain $\pi \leq \theta \leq 2\pi$.

d. Explain how to obtain the formula above for the function $f(\theta)$ on $(0, \pi)$. Hint: Sketch lots of examples.

e. Use MATLAB to create and print a graph of $f(\theta)$. (printed graph to be submitted separately to professor.)



A wedge of pizza. (Yum!)