MTH229

Graphical Solutions to Equations

Project 4– Exercises

NAME: ______________________________
SECTION: ______________________________
INSTRUCTOR: ______________________________

Exercise 1:
Use a graph of $(x-2)^2 = 4\sin(x)$ to find solutions to the equation valid to 2 decimal points:
(1) Answer: ________________________

Exercise 2:
Use the zooming technique to find solutions of

$$50 + \sin x = 2x.$$

which are valid to at least two decimal places.
Hint: Try to estimate the value of $50 + \sin x$*. This will give you an idea in which x interval are the possible solutions!*
(2) Answer: ________________________

Exercise 3:
Folklore is that exponential functions grow faster than polynomial functions. Although true, you need to be careful about how you interpret this statement, as this exercise shows.

Consider the functions $z_1 = e^x$ and $z_2 = x^4$. Plot them together on the interval [0,4].

a. From their graphs, how can you determine which graph is the exponential and which is the polynomial?
   (3) Circle one:
   1. polynomial functions grow faster than exponential functions
   2. Exponential functions grow faster than polynomial functions
   3. For different values of x, I can evaluate z_1, z_2 and determine which is larger.

b. Find the value of x (to two decimal places) for the point of intersection by zooming on the zero of $f(x) = e^x - x^4$. (or by zooming on the intersection point of the functions $z_1 = e^x$, $z_2 = x^4$.)
(4) Answer: ______________________________

On this graph, x^4 is larger than e^x from the intersection point to $x = 4$. Experiment to determine how large a value of x is needed for the exponential to catch up to x^4. Then find the second intersection point. (correct to three decimal places.) This one is larger than 4. In fact, you now have found two intersection points (x_1, y_1), (x_2, y_2). (where $x_1 < x_2$) Up to x_1 the function e^x is bigger, from x_1 to x_2 the function x^4 is the bigger. What happens after x_2?

c. What is the x-coordinate of the second intersection point?
(5) Answer: ______________________________

d. What happens to the behavior of z_1 and z_2 after the second intersection point?
(6) Circle one:
1. e^x grows faster
2. x^4 grows faster
3. they grow at the same rate
4. e^x grows faster, but for increasingly large values of x, x^4 catches up to e^x again.

Exercise 4:

a. Find the x-coordinate for where $f(x) = (x + 2)/x^2$ achieves its minimum value.
(7) Answer: ______________________________

b. What interval on the x-axis did you use to make you plot window?

(8) Answer:

Exercise 5:

a. Let $f(x) = x^3 - 7x^2 + 2x + 9$. Solve the cubic equation $f(x) = 0$. Find all of its roots correctly up to 4 significant digits.
(9) Circle one:
1. 6.6 , 1.1 -0.7
2. 6.4766, 1.4692, -0.9458
3. 6.7053 , 1.3259 , -0.8259
4. 0.0010, 1.0100, 7.5902
5. 6.5806 , 1.1062, -0.6868

b. Now find all solutions to $x^3 + 2x + 4 = 0$ (Note that the coefficient of x^2 is now 0).
(10) Circle one:
1. 0.6641, -0.6640, -1.3283
2. 1.8230, -1.8230, -1.3283
3. $0.5898 \pm 1.7445i \quad -1.1795$
4. $1.8230 \pm 0.6641i$, -1.3283

Exercise 6:

a. Let $\theta = \pi/4$. Look carefully at $f(x)$, it is a quadratic polynomial in x. Rewrite $f(x)$ so that the coefficients appear as (careful with the scientific notation)

$$f(x) = ax^2 + bx + c.$$

Now represent this polynomial in MATLAB, as in `[a b c]`. What are the values:

(11) Answer:

b. Use your previous answer and the `roots` function to find the range ($[0, b]$) of an arrow when shot at an angle of $\pi/4$. Specify the range in terms of its endpoint b.

(12) Answer: ______________________________

Exercise 7:

a. Plot various graphs of the $g(x)$ until you find the range of g, $[0, b]$. Enter the value of b with at least 1 digit to the right of the decimal point. (Remember, arrows don't bounce up – this mathematical model is only valid until the arrow first hits the ground.)
(13) Answer: ______________________________

b. Now make a plot containing the trajectories of both models. Label the individual plots.
(14) **Attach your graph to the worksheet.**

c. From your graph estimate the maximum height of the arrow if there is no wind resistance.
(15) Answer: ______________________________

d. From your graph estimate the maximum height of the arrow if there **is** wind resistance.
(16) Answer: ______________________________