MTH229

Investigating Limits in MATLAB

Project 5– Exercises

NAME: ______________________________
SECTION: ______________________________
INSTRUCTOR: ______________________________

Exercise 1:
Use the graphical approach to find the following **right** limit of $f(x) = x^x, \quad x > 0$

$$\lim_{x \to 0^+} x^x$$

What is the value of the limit?
(1) Answer: ______________________________

Exercise 2:

a. Use the numeric approach to find the following limit

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2}$$

What is the value of the limit?
(2) Answer: ______________________________

b. Numerically investigating a limit at a point c which is not zero requires us to use points which get close to c. A simple trick allows us to reuse our points which get close to 0. For instance if

```
>> x = [.1 .01 .001 .0001 .00001]; % and
>> c = 1/2;
```

Then we can make points getting close to c from above and below as follows:

```
>> c + x;                              % points above c
>> c - x;                              % points below c
```

Use this to investigate

$$\lim_{x \to \frac{\pi}{2}} (\frac{\pi}{2} - x) \tan(x).$$

What value for the limit do you get? (Enter no limit as DNE.)
(3) Answer: ______________________________

Exercise 3:
To illustrate what can happen numerically with some functions, we again look at finding the limit

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2}.$$

Only we let x get really close to 0. First we use some MATLAB shortcuts to define increasingly smaller values in x

```
>> n = 1:8; x = 10^(-n);          % Small values
```

Now produce a table of the $f(x)$ values for these values of x. Use this to investigate the limit at 0 of the function. How does the numerical instability of the problem show up in the output?
(4)

Exercise 4:
Let $f(x) = x \ln(x), \quad x > 0$. Calculate the following right limit with MATLAB as directed:

$$\lim_{x \to 0+} f(x).$$

a. Use a vector of numbers that converge to 0 from the right to investigate numerically the limit as x goes to 0. What is your numeric estimate for the limit?
(5) Answer: ______________________________

b. Enter in your table of values here. (Use the syntax `[x;y]'`.)
(6)

c. Plot a graph of $f(x)$ over $(0, 1)$. Does the graph confirm your limit found from the table of numbers?
(7) Circle one:
1. Yes **2.** No

d. Although the function is only defined for $x > 0$, plot a graph of $f(x)$ over $(-1, 1)$. Does the graph show anything unusual?

(8)

Exercise 5:
Plot the function $f(x) = x \sin(1/x)$, over the interval $[-\pi, \pi]$ using 100 points for x.

a. Try to `zoom` in on the limit. What happens near $x = 0$ that prevents you from getting the correct answer?
(9)

b. Graph the same function using 1000 points on top of your current graph (`hold`) using a different color.

c. The graph with 100 points didn't allow us to zoom in far enough to see the limit as this function oscillates so often that when we zoom in we eventually get just straight lines.

Do you have this same problem using 1000 points?
(10) Circle one:
1. Yes
2. No

d. Motivated by the squeeze theorem, on the same graph (`hold on`) plot both $y = |x|$ and $y = -|x|$ ($|x|$ is `abs(x)` in MATLAB) What relationship, if any, can be observed between $f(x) = x \sin(1/x)$ and these two lines?

(11) Circle all that apply:
1. $|x| \leq f(x)$
2. $|x| \geq f(x)$
3. $-|x| \leq f(x)$
4. $-|x| \geq f(x)$

e. Graphically estimate the limit as $x \to 0$ What is the limit?
(12) Answer: ______________________________

f. How did graphing the absolute value functions help you find the limit?
(13) Circle one:
1. The mean-value theorem
2. The function is continuous
3. the squeeze (or sandwich) theorem
4. they didn't I just guessed

Exercise 6:
We wish to find the right limit of the function $f(x) = x^{-1}e^{-1/x}$ as x goes to 0. (Notice it is indeterminate of the form $\infty \cdot 0$).

a. Write a function m-file to evaluate this function. Call the function `f.m`
(14)

b. Plot the function $f(x)$ over the interval $[0, 1]$ with these commands (using the definition of `f`)

```
>> x = linspace(0,1)
>> plot(x,f(x))
```

Use your graph to estimate the **right** limit as x goes to 0.
(15) Answer: ______________________________

Exercise 7:
Write a function `m`-file to evaluate the function $g(x) = x^x$, even if x is a vector of numbers. Use the name `g` for this function so that both `f` and `g` may be referred to below.

What is your function template? (16)

Exercise 8:

a. Find the following limit using composition of functions:

$$\lim_{x \to 0+} g(f(x))$$

What is the value of the limit?
(17) Answer: ______________________________

b. There are many consequences of the definition of limits that make finding limits easier. For instance, the limit of a sum is the sum of the limits (provided all the limits exist). Another is the limit of compositions can be expressed in terms of the limits. That is, if

$$\lim_{x \to c} f(x) = L, \quad \text{and } \lim_{x \to L} g(x) = M,$$

then

$$\lim_{x \to c} g(f(x)) = M.$$

Verify that this is the case with f and g defined as in the last problem. Is it true?
(18) Circle one:
1. yes **2.** no

Exercise 9:
The derivative of a function $g(x)$ at x is given by the following limit

$$\text{The derivative at } x = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

Let $g(x) = x^x$. This should be defined in a function m-file called `g`. If not, do so now.

a. Find the derivative of $g(x)$ at $x = 1$ by investigating the above limit. Use the expression

```
>> y = (g(1+h) -g(1)) ./h
```

rather than doing the composition and secant line expression directly from the function definition. What is the value of the limit?
(19) Answer: ______________________________

b. Now we try to to find the *right* limit of the derivative formula when $x = 0$. As $g(0)$ is not defined, we replace it with it's limit of 1.

What do you find when investigating

$$\lim_{h\to 0+} \frac{g(h) - 1}{h}?$$

(20)