MTH229

Approximate First and Second Derivatives

Project 6– Exercises

NAME: __________________________________
SECTION: __________________________________
INSTRUCTOR: __________________________________

Exercise 1:
Let $f(x) = \sin(x^2)$. We wish to find the derivative of f when $x = \pi/4$.

a. Make a function `m`-file storing the function $f(x) = \sin(x^2)$ with name `f.m`. What are the contents of the file:
(1)

b. Take $h = 0.1$ and find the slope of the secant line at $(x, f(x))$:

$$\frac{f(\frac{\pi}{4} + h) - f(\frac{\pi}{4})}{h}.$$

(2) Answer: ______________________________

c. Take smaller values of h to find a better approximation to $f'(\pi/4)$ (Four decimal points of accuracy is good.)
(3) Answer: ______________________________

Exercise 2:
We will see how the secant line approaches the tangent line for the function $f(x) = \sin(x)$ at the point $x_1 = \pi/4$. Do this by following this sequence of steps. When you are done you will upload your graph to show your work.

A secant line is just a line drawn between two points on the graph of $f(x)$, where one of the points is the point of interest like $x_1 = \pi/4$ in this example. The second point can be called x_2, but we prefer to emphasize its relationship to x by writing $x_2 = x + h$ where h then is the distance between the two points.

a. Make a function m-file storing the function $f(x) = \sin(x)$ with name `f.m`. What are the contents of the file:
(4)

b. Graph $f(x)$ over the interval $[0, \pi]$.

c. Let $h = \pi/2$. On your graph, add the secant line connecting the points $x_1 = \pi/4$ and $x_2 = x_1 + h = 3\pi/4$.

The following commands will help you plot such a line.

```
>> x1=pi/4;
>> h = pi/2;  x2 = x1+h; m = (f(x1+h)-f(x1)) / h ;
>> plot(x, f(x1) + m*(x-x1))                % point-slope form of line
>> plot(x2,f(x2),'*')                       % mark the point
```

What is the slope of this secant line?
(5) Answer: ______________________________

d. For $h = 0.1$, and $h = 0.001$ draw the secant line as above. Label the different secant lines with the value of h.

e. The tangent line at $x_1 = \pi/4$ has slope $\sqrt{2}/2$. Plot the tangent line on the same graph in a different color and label it.

f. Submit the graph of Example 1.

(6) **Attach your graph to the worksheet.**

g. Explain carefully the relationship between h and slope of the secant lines for h. In particular, what is the limit of these slopes as h gets closer to 0?

(7)

Exercise 3:

Have a close look at the resulting graphs of Example 2. Identify which is the graph for `cos(x)` and which is the graph for `difquo`.

a. Does the graph of `difquo` resemble the graph of a function that you are familiar with?
   (8) Circle one:
   1. $\sin x$
   2. $-\sin x$
   3. $\cos x$
   4. $-\cos x$

b. Based on your answer to part 1, what do you think is the derivative of $y = \cos x$?
   (9) Circle one:
   1. $\sin x$
   2. $\cos x$
   3. $-\cos x$
   4. $-\sin x$

Exercise 4:

It is known that the derivative of the function $f(x) = x^{3/2} + 5$ is given by $f'(x) = 3/2\sqrt{x}$. We will verify that the difference quotient converges to this function as h goes to 0 over the interval $[0, 2]$

a. Write an `m`-file storing the definition of $f(x)$. List its contents here:
   (10)

b. For $h = 0.1$ make a graph of the difference quotient and the derivative. What is the largest difference between the two functions?
   (11) Answer: ____________________________

c. On the same graph, plot the difference quotient for $h = 0.001$. What is the maximum difference now between the difference quotient and the derivative function?
   (12) Answer: ____________________________

Exercise 5:
Numerical differentiation of exponential function

You may not know the derivative of $f(x) = e^x$ and $g(x) = \ln(x)$ at this point. Let's use the difference quotient to figure out the derivative of e^x.

a. Use MATLAB to create a continuous plot of $f(x) = e^x$ over the interval $-2 \leq x \leq 2$. Based on this graph, which of the following should be true about the derivative.
(13) Circle all that apply:
1. $f(x)$ is always increasing so $f'(x) > 0$.
2. Tangent lines for $f(x)$ are never 'flat' so there are no critical points.
3. $f(x)$ increases faster and so $f'(x)$ should be increasing

b. On top of the graph of $f(x)$, create a plot of the difference quotient of $f(x)$ using $h = 0.01$ over the same interval $[-2, 2]$ in a different color. Turn in your graph.
(14) **Attach your graph to the worksheet.**

c. Based on the two plots, what do you think the derivative of $f(x) = e^x$ is.
(15) Circle one:
1. x^e
2. $\log x$
3. x^2
4. e^x

Exercise 6:
We wish to graph the function $f(x) = e^{x/3} \sin(\pi x)$.

a. Write an m-file that evaluates this function. Call your m-file `f.m`.
(16)

b. Make a plot of $f(x)$ over the interval $[0, 3]$.

c. Using $h = 0.01$ compute the difference quotient for x in the interval $[0, 3]$. Write down the commands to find the difference quotient and add its graph to the graph of $f(x)$.
(17)

d. Again using $h = 0.01$, find the approximate second derivative (`difdifquo`) and add its plot to your plot of the function. What are the commands you used to find `difdifquo`?
(18)

e. Use an arrow to label any zeroes of the approximate first derivative on your graph. What can you say about the graph of $f(x)$ near these x values?
(19)

f. Submit your three graphs in one figure with the requested annotations.
(20) **Attach your graph to the worksheet.**