MTH229

Critical and Inflection Points

Project 7– Exercises

NAME: _____
SECTION: _____
INSTRUCTOR: _____

Exercise 1:

a. Find the zeros of $f'(x)$ correct to two decimal places.
(1) Answer: _____

b. How do the zeros of $f'(x)$ relate to $f(x)$?
(2) Circle one:
1. They do not relate to $f(x)$
2. they're the values for which f(x) has its critical points
3. they're the values for which f(x) = 0

c. From its graph, find the intervals on which $f'(x)$ is negative?
(3) Circle one:
1. (0,0.75) , (1.5,∞)
2. (-∞,0) , (0.75,1.5)
3. (-∞,0) , (0.74,1.2)
4. not listed

d. What can you determine about $f(x)$ on these intervals?
(4) Circle one:
1. $f(x)$ increases where $f'(x) > 0$ and decreases where $f'(x) < 0$
2. $f(x)$ increases where $f'(x) < 0$ and decreases where $f'(x) > 0$
3. $f'(x)$ increases where $f(x) > 0$ and decreases where $f(x) < 0$
4. none of the above

e. Use the zeros of $f'(x)$ to find the coordinates of the relative maximum of $f(x)$ correct to two decimal places. (Note: After finding the proper value of x let the computer calculate the corresponding value of y. First enter the value of x. Recall y with the up-arrow key and enter. Then type y and press the enter key).

f. The coordinates of the relative maximum are:
(5) Circle one:
1. (0.63,1.76) and (1.27,4,78)
2. (0.55,2,43)
3. (0.75,1.27)
4. not listed

g. Find the coordinates (both x and y), correct to two decimal places, of the absolute maximum and the absolute minimum of $f(x)$ on the interval [0,2].

h. The absolute maximum is:
(6) Circle one:
1. (2,4) endpoint
2. (2,4) critical number
3. (1.5,1.2) critical number
4. not listed

i. The absolute minimum is:
(7) Circle one:
1. (1,0) critical number
2. (1.2,.3) critical number
3. (0,0) and (1.5,0) both critical numbers
4. not listed

Exercise 2:
To find the points of inflection, we examine the zeros of $f''(x)$, the second derivative. We use the name `ypp` for this.

```
>> hold on
>> ypp= 48*x.^2 - 72*x + 18;
>> plot(x,ypp,'g')
```

a. On what intervals is $f''(x)$ positive?
(8) Circle one:
1. (-3,1.83)
2. (0.3170,1.1830) and (1.1830,∞)
3. (-∞,1.1830)
4. (-∞,0.3170) and (1.1830,∞)

b. On what intervals is $f''(x)$ negative?
(9) Circle one:
1. (0.3170,1.1830)
2. (0.3170,1.1830) and (1.1830,∞)
3. (-∞,1.1830)
4. (-∞,0.3170) and (1.1830,∞)

c. What is the relationship between the sign of the function $f''(x)$ and the concavity of $f(x)$.
(10) Circle all that apply:
1. $f''(x) > 0$ on [a,b] implies that $f(x)$ is concave down on $[a, b]$
2. $f''(x) < 0$ on $[a, b]$ implies that $f(x)$ is concave up on $[a, b]$
3. $f''(x) < 0$ on $[a, b]$ implies that $f(x)$ is concave down on $[a, b]$
4. $f''(x) > 0$ on $[a, b]$ implies that $f(x)$ is concave up on $[a, b]$

d. Find the coordinates of the points of inflection of $f(x)$ correct to two decimal places by finding the zeros of $f''(x)$ graphically. Give both coordinates of each inflection point. Again, use $y = f(x)$ to calculate the y-coordinate of the inflection point.

The coordinates are: (*Be careful.*)
(11) Circle one:
1. (0.31,0.52) and (1.14,0.44)
2. (0.29,0.52) and (1.78,0.44)
3. (0.31,0.50) and (1.14,0.44)
4. (0.32,0.56) and (1.18,0.56)

e. Find $f''(x)$ and solve for its zeros using the quadratic equation. Find the x-coordinates of the inflection points and compare to the results found graphically.
The coordinates are:
(12) Circle one:
1. $2 - \sqrt{2}$ and $2 + \sqrt{2}$
2. $(3 + \sqrt{3})/4$ and $3/4$
3. $(3 - \sqrt{3})/4$ and $(3 + \sqrt{3})/4$
4. $(3 - \sqrt{3})/4$ and π

f. Attach a graph of $f(x)$, $f'(x)$ and $f''(x)$. Please identify the three functions on your graph.
(13) **Attach your graph to the worksheet.**

Exercise 3:
This exercise uses information about the derivative of a function to infer properties of the unknown function. While it is true that we can not completely reconstruct $f(x)$ from $f'(x)$ without some additional detail, we can say characterize for $f(x)$ its relative extrema and concavity.

Let the derivative of a function $f(x)$ be given by $f'(x) = x^3 - 7x^2 + 14$. We'll investigate the behavior of $f(x)$ given this information.

Graph both $f'(x)$ and $f''(x)$ on the interval $[-4, 8]$ with a grid using a different color for each. Add a title with your name, and label the graphs in some manner.

We will use both graphs to answer the following questions about the unknown function $f(x)$.

(**Note:** Since in this exercise $f'(x)$ and $f''(x)$ are polynomials, instead of `zoom`ing, you could use the MATLAB `roots` command to find their zeros accurately. For example to find the roots of the polynomial $x^2 - x - 1$ you use the representation `[1 -1 1]` for the polynomial and the command `roots([1 -1 -1])` to find the roots.)

a. On what subinterval(s) is $f(x)$ increasing?
(14) Circle one:
1. (-1.29,1.61) and (6.69,∞)
2. (-∞,0) and (4.67,∞)
3. (-∞,0)
4. (-∞,0) and (6.69,∞)

b. Which of the following conditions did you use to find the subinterval(s) on which $f(x)$ increases?
(15) Circle one:
1. $f'(x) > 0$ on these subintervals
2. $f''(x) > 0$ on these subintervals
3. $f'(x) < 0$ on these subintervals
4. $f'(x)$ is increasing on these intervals

c. Find the x-coordinates of all relative minima of $f(x)$.
(16) Circle one:
1. $x = 4.56$
2. $x = -1.29$ and $x = 6.69$
3. $x = 4.56$ and $x = 0$
4. $x = -1.29$ and $x = 1.61$

d. On what subinterval(s) is $f(x)$ concave up?
(17) Circle one:
1. $(-\infty, 0)$ and (4.67,∞)
2. (1.166,∞)
3. (-∞,1.167)
4. it's always concave down

e. Which of the following conditions did you use to find the subinterval(s) on which $f(x)$ is concave up?
(18) Circle one:
1. $f'(x) > 0$ on these subintervals
2. $f''(x) > 0$ on these subintervals
3. $f'(x) < 0$ on these subintervals
4. this can only be known by examining the graph

f. Find the x-coordinates of all points of inflection of $f(x)$.
(19) Circle one:
1. $x = 4.56$
2. $x = 4.65$
3. $x = 14/3$ and $x = 0$
4. not listed

g. Submit your graph of $f'(x)$ and $f''(x)$.
(20) **Attach your graph to the worksheet.**

Exercise 4:

a. Use MATLAB to graph $f(x) = x - \sin(x)$ on the interval $[0, 4\pi]$. Use "hold on" to graph $f'(x)$ and $f''(x)$ on the same plot. Use MATLAB to label the three functions.
(21) **Attach your graph to the worksheet.**

b. Does $f(x)$ have any relative extrema on this interval? (Recall a relative extrema requires an open interval.)
(22) Circle one:
1. at $x = 0, 2\pi, 4\pi$
2. at $x = \pi/2$ $x = 5\pi/2$
3. none
4. at $x = \pi/2$ $x = 2\pi$

c. Give an explanation for your previous answer.
(23) Circle one:
1. these are the values where $f'(x) = 1 - \cos(x) = 0$
2. these are the values where $f'(x) = 1 - \cos(x) > 0$
3. these are the values where $f''(x) = \sin(x) = 0$
4. There are no values where $f'(x)$ changes sign

d. Identify all points of inflection. in $(0, 4\pi)$ (Do not include the endpoints.)
(26) Circle one:
1. (π, π)
2. $(\pi, \pi), (2\pi, 2\pi), (3\pi, 3\pi)$
3. $(\pi, \pi), (3\pi, 3\pi), (4\pi, 4\pi)$
4. $(\pi, \pi), (2\pi, 2\pi), (3\pi, 3\pi), (4\pi, 4\pi)$

Exercise 5:

The concentration f of a certain medicine in the bloodstream t hours after injection into muscle tissue is modeled by:

$$f(t) = \frac{3t^2 + 1}{50 + t^3}, \quad t \geq 0$$

Use the graphical capability of MATLAB to investigate the model.

a. Make a graph of the concentration, $f(t)$ for $t \geq 0$. (You need to decide how large t should be to answer the questions below.) On the same graph plot $f'(t)$ and $f''(t)$ or the approximate first and second derivatives, `difquo` and `difdifquo` of a previous project. Label the graphs. (It is easier to plot `difquo` and `difdifquo` as the derivative gets messy. For example if `t` has already been defined and you created a function m-file `f.m` then

```
>> h=.01;plot(t,(f(t+h)-f(t))/h, t, (f(t+h)-2*f(t)+f(t-h))/h^2))
```

will plot them.)

(27) **Attach your graph to the worksheet.**

b. When will there be maximum concentration?

(28) Answer: _____

c. How much is the maximum concentration?
(29) Answer: _____

d. When will the concentration dip below a level of 0.1?
(30) Circle one:
1. $t = 30$
2. $t = 40$
3. $t = 50$
4. $t = 55$

e. Estimate graphically where the concentration function changes concavity?

(31) Answer: _____

f. In this model, is the concentration ever zero?

(32) Circle one:
1. yes **2.** no