MTH229

Newton's method

Project 8– Exercises

NAME: _____
SECTION: _____
INSTRUCTOR: _____

Exercise 1:
Evaluate $f(x)$ at the value for x_8 displayed in the example, `x=1.4422`. Is $|f(1.4422)| < 10^{-5}$?
(1) Circle one:
1. yes **2.** no

Exercise 2:
Let's use Newton's method to find the value of $\sqrt{2}$. (This is a zero of the equation $f(x) = x^2 - 2 = 0$.)

- Start with an initial guess of $x_0 = 3$ and use Newton's method to approximate the zero, $\tilde{x}$. What is the answer?
  (3) Answer: _____

- How many iterations did it take?
  (4) Answer: _____

- What is the value of $f(\tilde{x})$?
  (5) Answer: _____

- Print out a graph of $f(x) = x^2 - 2$ on the interval $(-4, 4)$. Using a straight edge (another piece of paper or a ruler) graphically do Newton's method on your printout starting with a value of $x_0 = 3$ and $x_0 = -1$. Do you get the same root each time?
  (6) Circle one:
  1. yes **2.** no

Exercise 3:
Apply Newton's method to find a root of $f(x) = x^3 + 2x^2 - 30x - 5$.

a. First, plot the function $f(x) = x^3 + 2x^2 - 30x - 5$ over the interval $(-5, 5)$. How many times does the graph cross the x-axis? (Why can't it be more than 3?)
(7) Circle one:
1. 1
2. 2
3. 3
4. 4

b. Find the largest root using Newton's method starting at $x_0 = 4$. It should be clear that the largest root has a value between 4 and 5 from your graph. Zero in on this root. What value do you find? (use `format short`)
(8) Answer: _____

c. How many iterations did it take?
(9) Answer: _____

d. Check that your value of $f(\tilde{x})$ is close to 0. If it is not, you made a mistake.

(Of course you could check this answer with the `roots` command.)

Exercise 4:
Let $f(x) = e^x - x^4$. (Then $f'(x) = e^x - 4x^3$.) This function has three zeroes in the interval $[-1, 10]$.

a. What zero is found from Newton's method when it is started with $x_0 = -1$?
(10) Answer: _____

b. What zero is found from Newton's method when it is started with $x_0 = 7$?
(11) Answer: _____

c. What zero is found from Newton's method when it is started with $x_0 = 8$?
(12) Answer: _____

d. What zero is found from Newton's method when it is started with $x_0 = 7.3$?
(13) Answer: _____

Exercise 5:
Solve the equation

$$\sin(x) = x/4,$$

using Newton's method starting with $x_0 = 2\pi$. Figure 1 illustrates the algorithm graphically.

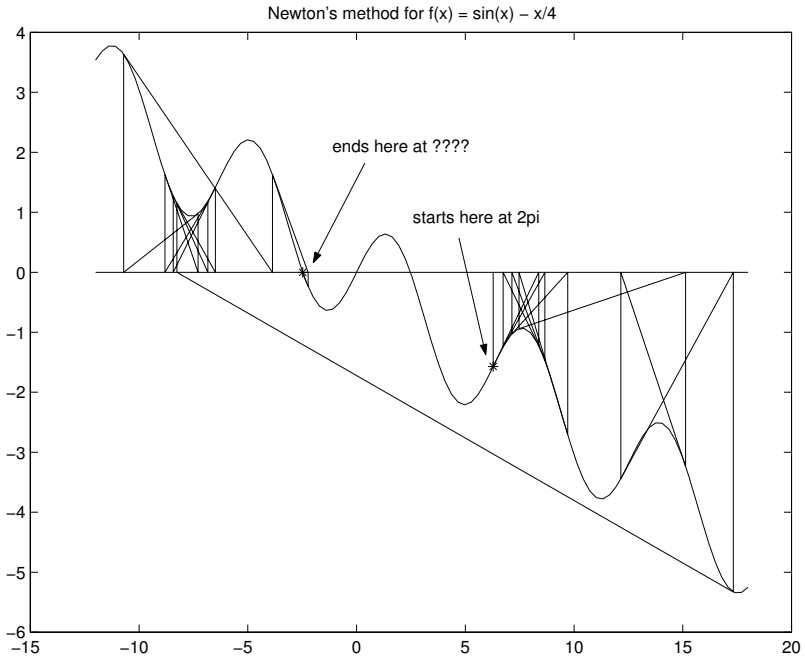

Figure 1: Example of Newton's method where convergence is not as expected due to poor choice of initial point.

a. What is the value returned by Newton's method for the zero?
(14) Answer: _____

b. How many iterations did it take?
(15) Answer: _____

Exercise 6:

a. Factor $x^3 - 5x$ to find the exact zeroes. How many are there?
(16) Circle one:
1. 0
2. 1
3. 2
4. 3

b. What are they? (enter square roots as `sqrt(x)`)
(17) Answer:

c. Let the initial guess be 1. Use Newton's method to try and find a zero. What happens? (be specific)
(18)

d. Does this same behavior occur with $x_0 = 2$? What answer does Newton's method give for $\tilde{x}$ in this case?
(19) Answer: _____

To fully understand what is going on with this example, trace a few iterations of Newton's method with your finger on your graph.

Exercise 7:
Next we study the behavior of Newton's method for the function $f(x) = x^{1/3}$. This problem can be done algebraically (and in fact, doesn't work numerically for some versions of MATLAB) so we ask it to be done by hand.

a. Write out the equation $x_{n+1} = x_n - f(x_n)/f'(x_n)$ and simplify the right hand side **using algebra**. Write your answer in MATLAB notation. (*Don't be surprised if this problem simplifies a lot*)
(20) Answer:

b. Starting with $x_0 = 1$ find the values of x_3 and x_4.
First, $x_3 =$?
(21) Answer: _____

And $x_4 =$?
(22) Answer: _____