MTH229

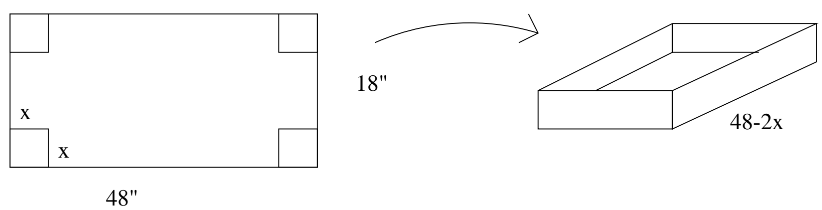
Optimization

Project 9– Exercises

NAME: ______________________
SECTION: ______________________
INSTRUCTOR: ______________________

Exercise 1:
Dimensions of the Largest Box

An open box is to be made from a rectangular piece of cardboard measuring 18" × 48". The box is made by cutting equal squares from each of its 4 corners and turning up the sides. (Suggestion: you can try making one yourself with a spare piece of paper.)



a. Let x be the side of a square removed from each corner. Express the volume V of the box as a function of x. (The length is $48 - 2x$, what is the width?)
$V(x) = ?$
(1) Answer:

b. What is the domain of possible values for x? (That is, the domain of $V(x)$?)
(2) Answer:

c. Make a graph of $V(x)$ over the domain chosen. Graphically determine all possible values of x so that the volume of the box is $V = 1400$ cubic inches. Determine these values to an accuracy of 3 significant digits.
(3) Answer: ______________________________

d. The equation $V(x) = 1400$ is a cubic equation with 3 real roots. Find all three roots using MATLAB's "roots" command. Do you get the same answers as your graphical investigation. Explain.
(4)

e. Graph $V(x)$ and $V'(x)$ together. Use the graph of $V'(x)$ to find the value of x that maximizes V.
(5) Answer: ______________________________

f. Find $V'(x)$ analytically by differentiating your formula for $V(x)$. Does your formula show that your last answer is a critical point for $V(x)$?
(6) Circle one:
1. yes **2.** no

g. Find the exact value of the maximum of $V(x)$.
(7) Answer: ______________________________

Exercise 2:
Largest rectangle inscribed in a semicircle

Determine the area of the largest rectangle that can be inscribed in a semicircle of radius 8". Figure 1 shows that the area can be written as $A = (2x)y$, if (x, y) is the point of the upper right corner of the rectangle. However, we choose to *parameterize* the area by a single value, the angle θ.

a. Derive the formula for the area of the inscribed rectangle as a function of θ. We refer to this function as $A(\theta)$ below.

$A(\theta) =?$

(8)

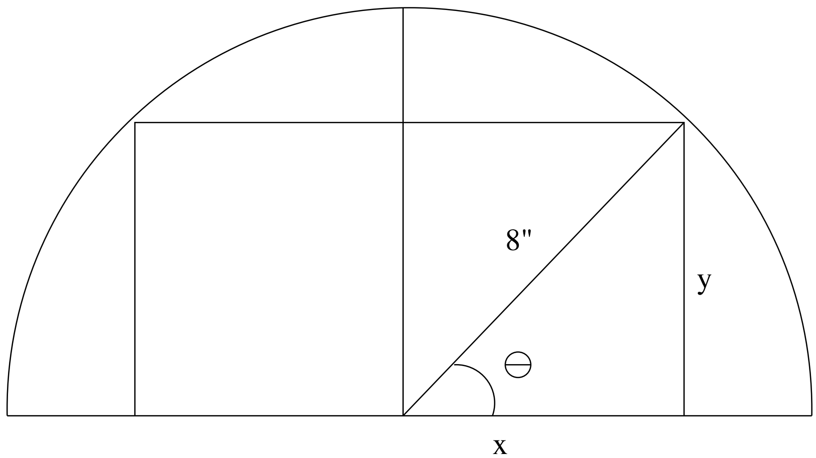


Figure 1: What is the rectangle with largest area that can be inscribed in a half circle?

b. Plot $A(\theta)$ over its relevant domain. What is the relevant domain?

(9) Circle one:
1. $[0, \pi]$
2. $[0, \pi/2]$
3. $[0, 8]$
4. $[0, 4]$
5. None of these answers

c. For what value of θ is the maximum area attained?
(10) Answer: ______________________________

d. What is the maximum area?
(11) Answer: ______________________________

Exercise 3:
A walk in the park?
Eva wants to get to the bus stop as quickly as possible. The bus stop is across a grassy park, 2000 feet west and 600 feet north of her starting position. Eva can walk west along the edge of the park on the sidewalk at a speed of 6 ft/sec. She can also travel through the grass in the park, but only at a rate of 5 ft/sec (the park is a favorite place to walk dogs, so she must move with care). What path will get her to the bus stop the fastest? Remember,

$$\text{distance} = \text{rate} \times \text{time}.$$

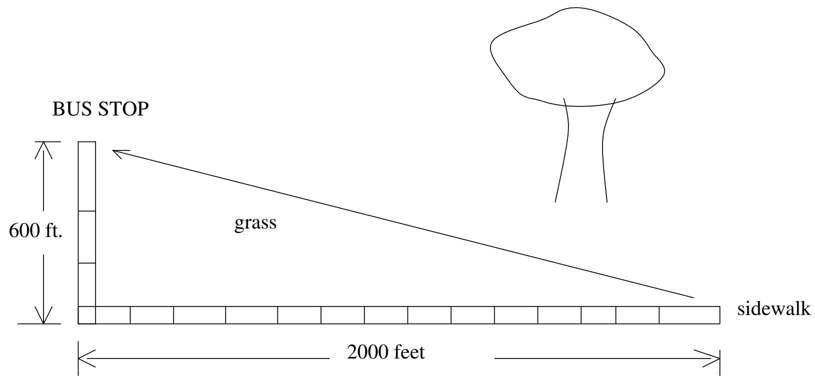


Figure 2: What is fastest way to walk to the bus stop?

a. How long would it take Eva to walk to the bus stop if she doesn't want to get her new shoes dirty? (If she only walked on the sidewalk.)
(12) Circle one:
1. $t = 2600/6$
2. $t = 2600/5$
3. $t = \sqrt{2000^2 + 600^2}/6$
4. $t = \sqrt{2000^2 + 600^2}/5$
5. None of these answers

b. How long would it take Eva to walk to the bus stop if she took the route of shortest distance? (A diagonal through the park.)
(13) Circle one:
1. $t = 2600/6$
2. $t = 2600/5$
3. $t = \sqrt{2000^2 + 600^2}/6$
4. $t = \sqrt{2000^2 + 600^2}/5$
5. none of these answers

c. If Eva walks x feet west along the sidewalk and then walks diagonally through the park, which function below represents the time it will take her to follow this path?

(14) Circle one:
1. $t = x/6 + \sqrt{2000^2 + 600^2}/5$
2. $t = x/5 + \sqrt{2000^2 + 600^2}/6$
3. $t = x/6 + \sqrt{(2000 - x)^2 + 600^2}/5$
4. $t = x/6 + \sqrt{2000^2 + (600 - x)^2}/5$
5. none of these answers

d. Use your answer in part 3 above and MATLAB to determine the shortest time it could take Eva to get to the bus stop. The minimum time is:
(15) Circle one:
1. 360
2. 400
3. 433
4. 417
5. none of these answers

e. How far should Eva walk west along the sidewalk in order to minimize the amount of time it takes her to get to the bus stop?
(16) Circle one:
1. 0
2. 400
3. 1,100
4. 2,000
5. none of these answers

Exercise 4:
The ladder problem
A two-dimensional contractor would like to take a ladder down a hallway, but must negotiate a corner. The dimensions of the hallway are illustrated in Figure 3. That is, one width is 8 feet, the other 5 feet. What is the longest ladder that the contractor can successfully get around the corner and through the hallway?

Figure 3 also has drawn on it the longest possible ladder that can fit for a given angle t. This ladder will touch in three points: the corner, and the two outside edges of the hallway.

We focus on the function $l(t)$ which gives the length of the longest ladder that can fit for a given angle t.

a. Find a formula for $l(t) = x + y$:
(17) Circle one:
1. $l(t) = 5\tan(t) + 8\cot(t)$
2. $l(t) = 5/\cos(t) + 8/\sin(t)$
3. $l(t) = 5\cos(t) + 8\sin(t)$
4. $l(t) = (5^2 + \cos(t))^{1/2} + (8^2 + \sin(t))^{1/2}$

b. What is the length of the longest ladder that can fit when the angle is $\pi/6$ radians (30 degrees)?
(18) Answer: ______________________________

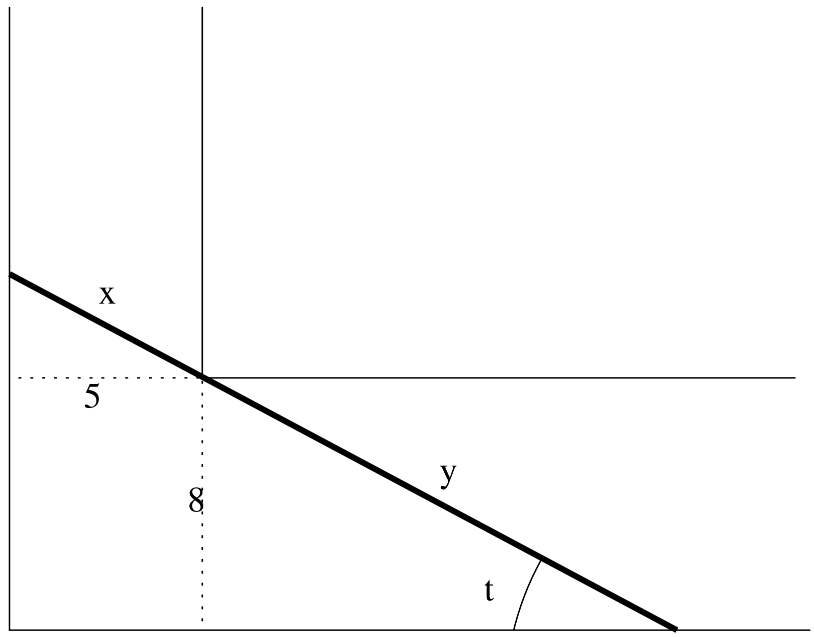


Figure 3: Illustration of the longest ladder, $l(t) = x + y$, that can fit at an angle t.

c. A 20 foot ladder will get stuck on its way around the corner of the hallway – at one angle if carried through the 8' hallway, and at another angle through the 5' hallway. What are these two angles?
(19) Answer: ________________________________

d. Plot a graph of $l(t)$ over a reasonable viewing window. Find its minimum value and locate the t value for which this happens. What is the angle t? (Answer in radians)

(20) Answer: ________________________________

e. If a ladder is longer than this minimum value, there will be angles for which it won't fit around the corner. For ladders shorter than this minimum value this won't be the case. Use this to find the length of the longest ladder the contractor can carry around the corner.

The longest ladder is:
(21) Answer: ________________________________