MTH229

Definite Integrals and Riemann Sums

Project 10– Exercises

> NAME: _____
> SECTION: _____
> INSTRUCTOR: _____

Exercise 1:

a. Create a script `m`-file containing the content above, only change the value of n to 2,500. What is the area for the Riemann sum now?
   (1) Answer: _____

b. The advantage of script `m`-files is that they can be easily edited and run. Simply change what you want, save your work, and call it from the command line. Try all of these values of n: 7,500, 15,000 and 30,000 until you find a value where the Riemann sum is 1 to 4 decimal points. What value of n do you find?
   (2) Circle one:
   1. 7,500
   2. 15,000
   3. 30,000

Exercise 2:
We will now compare the area under $\sin(x)$ to that under $f(x) = \sin(\sqrt{x})$. Figure 1 shows graphs of both functions over $[0, \pi]$.

```
>> x = linspace(0,pi); plot(x,sin(x),x,sin(sqrt(x)));grid
```

We see that the area under the $\sin(\sqrt{x})$ curve looks larger than 2 — the area under $\sin(x)$. We find the actual area to be certain. We don't know an anti-derivative to this function so the fundamental theorem of calculus will be of no help. Rather, use the Riemann sum approximation to find the area associated to the definite integral
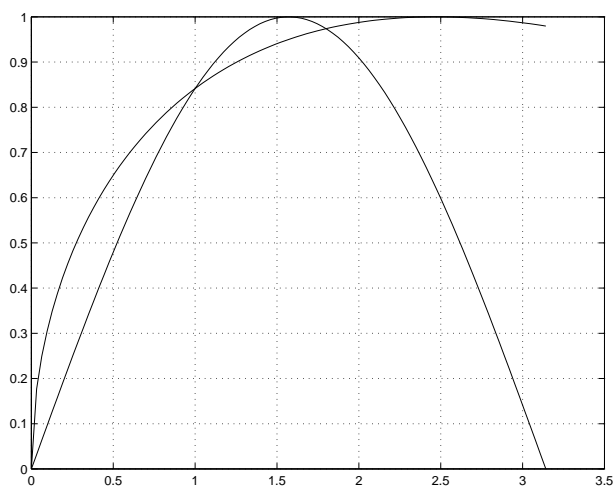
$$\int_0^\pi \sin(\sqrt{x})dx.$$

Figure 1: Graphic comparing area under $\sin(x)$ and $\sin(\sqrt{x})$ between $[0, \pi]$.

a. Find the value of $\sum_{i=1}^{n} f(x_i)\Delta x$ for $n = 50$.
    (3) Answer: _____

b. Find the value of $\sum_{i=1}^{n} f(x_i)\Delta x$ for $n = 500$.
    (4) Answer: _____

Somewhere between $n = 25,000$ and $n = 30,000$ the value stabilizes to 4 digits, the value being 2.6695. Again, we see that this can be a slow process to converge.

Exercise 3:
The function $e^{-x^2/2}$ is very important in probability theory. Figure 2 shows the graph. To help estimate the area, two lines forming a triangle with the x axis are added

The triangle has area 2. From the graph then we know our answer should be slightly bigger than 2. Let's see what it is.

a. From the graph explain why

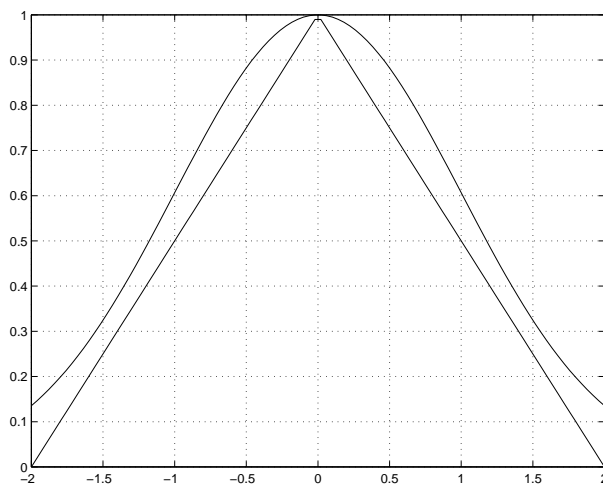$$\int_{-2}^{2} e^{-x^2/2}dx = 2\int_{0}^{2} e^{-x^2/2}dx$$

(5)

Figure 2: Graph of bell-shaped curve. Straight lines are added to estimate the area under the bell.

b. Use $n = 30,000$ to estimate using a Riemann sum

$$\int_{-2}^{2} e^{-x^2/2} dx.$$

(Here we have a different from 0.)
(6) **Answer:** _____

Exercise 4:
Find the approximate value of the integral

$$\int_{0}^{\pi} \frac{\sin x}{x} dx$$

by using a Riemann sum approximation with $n = 30,000$. (We avoid issues with the definition of the Riemann sum by assuming $f(0) = 1$, so that $f(x)$ is continuous.)
(7) **Answer:** _____

Exercise 5:
Many everyday vessels containing fluid are mathematically known as *surfaces of revolution*, as their surface can be created by rotating the graph of some function $f(x)$ around the x axis. For instance, the linear function $f(x) = 2.5 + x/10$, $0 \leq x \leq 15$ when rotated around the x axis, traces out a glass shape that is roughly the size of a 16-ounce tumbler. This is illustrated in Figure 3 with the x-axis running vertically instead of the more traditional horizontal direction.
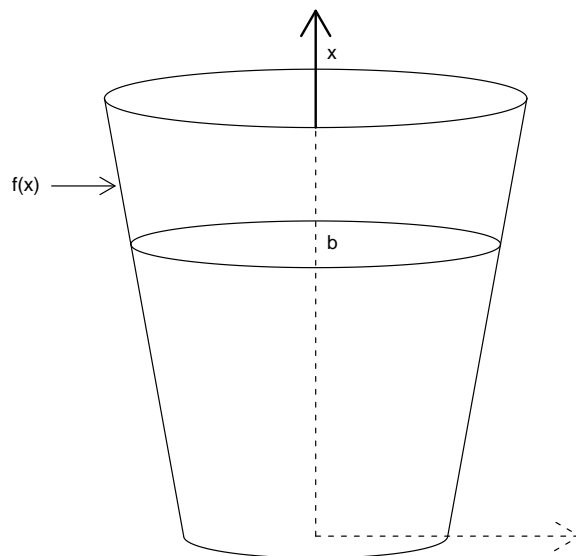
Figure 3: A tumbler described by $f(x)$ filled to a height of b.

The exact volume of fluid in the vessel depends on the height to which it is filled. If the height is labeled b, then the volume is

$$V(b) = \int_0^b \pi f(x)^2 dx.$$

a. Find the volume contained in the glass if it is filled to the top $b = 14$ cm. This will be in metric units of cm^3. To find ounces divide by 1000 and multiply by 33.82.

How many ounces does this glass hold?
(8) Answer: _____

b. Just what is meant by "the glass is half full?" If the glass is filled to $b = 7$ cm, what percent of the total volume is this?

Answer with a percent (Volume for 7/Volume for 14 times 100).
(9) Answer: _____

c. Now, by trying different values for b, find a value of b within 1 decimal point (eg. 7.4 or 9.3) so that filling the glass to this level gives half the volume of when it is full.

$b = ?$
(10) Answer: _____

Exercise 6:
The area under a curve $f(x)$ is given by the definite integral. But what about the length of the curve? This too is answered by a definite integral, but with a different formula:

$$\text{Length} = \int_a^b \sqrt{1 + (f'(x))^2}dx :$$

Let's use this formula to compute the length of the three curves shown in Figure 4.

a. Let $f(x) = x^2$. Find the length of the graph of $f(x)$ over the interval $[0, 2]$. (It should be between $4.472 = \sqrt{2^2 + 4^2}$ and $6 = 2 + 4$.)
(11) Answer: _____

b. Let $f(x) = e^x$. Find the length of the graph of $f(x)$ between $x = 0$ and $x = 2$. (It should be between $6.695 = \sqrt{2^2 + (e^2 - 1)^2}$ and $8.389 = 2 + (e^2 - 1)$.)
(12) Answer: _____

c. Let $f(x) = \ln(x)$. (This has derivative $1/x$.) Find the length of this curve from $1 = e^0$ to e^2. (The answer might be expected, given your last, after a bit of thought.)
(13) Answer: _____

Exercise 7:
Do the trapezoid method above for $n = 50$ to find an approximate answer for the definite integral
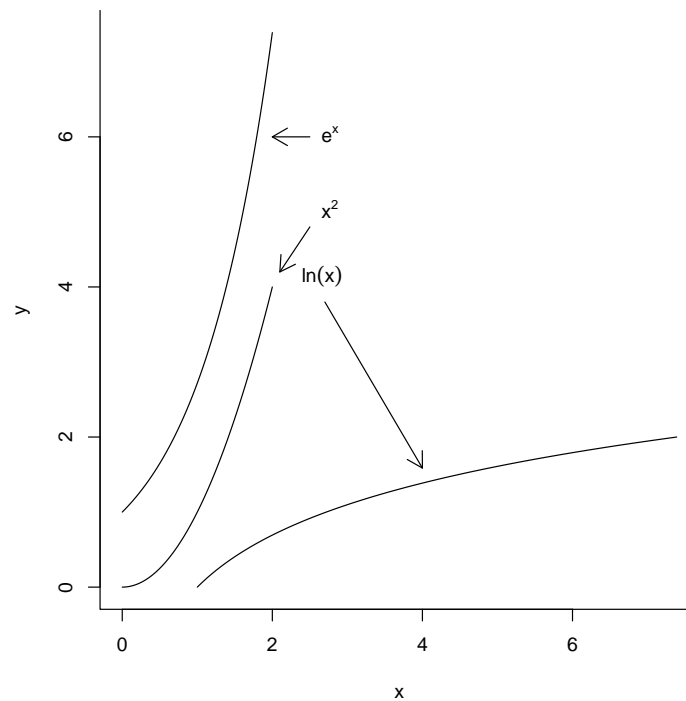
$$\int_0^\pi \sin(\sqrt{x})dx$$

a. **(14) Answer:** _____

Figure 4: What is the length of the three curves?

b. Has the method given the answer 2.6695 by the time $n = 1000$? (Remember it took an n between 25,000 and 30,000 using rectangles.)
    (15) Circle one:
    1. Yes **2.** No