MTH232

<div align="center">

Taylor Polynomials and Approximations

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Project 4– Exercises

NAME: _____

SECTION: _____

INSTRUCTOR: _____

Exercise 1:

We want to calculate some of the Taylor polynomials centered at $c = 0$ for $\cos x$, i.e. MacLaurin polynomials for $\cos x$, and we want to graph these functions on the same graph with $\cos(x)$ in order to view the error.

a.) Using the formulas above, compute $p_0(x)$, $p_1(x)$, $p_2(x)$, $p_4(x)$, for $f(x) = \cos(x)$ centered at $c = 0$. (You can do this without MATLAB)

- $p_0(x) =$
  (1) Circle one:
  1. $P_0(x) = 0$
  2. $P_0(x) = 1 - x^2/2$
  3. $P_0(x) = 1$
  4. $P_0(x) = 1 - x^2/2 + x^4/24$

- $P_1(x) =$
  (2) Circle one:
  1. $P_1(x) = 0$
  2. $P_1(x) = 1 - x^2/2$
  3. $P_1(x) = 1$
  4. $P_1(x) = 1 - x^2/2 + x^4/24$

- $P_2(x) =$
  (3) Circle one:
  1. $P_2(x) = 0$
  2. $P_2(x) = 1 - x^2/2$
  3. $P_2(x) = 1$
  4. $P_2(x) = 1 - x^2/2 + x^4/24$

- $P_4(x) =$
  (4) Circle one:
  1. $P_4(x) = 0$
  2. $P_4(x) = 1 - x^2/2$
  3. $P_4(x) = 1$
  4. $P_4(x) = 1 - x^2/2 + x^4/24$

Exercise 2:
Look at the graph of the functions of $\cos(x)$ and $P_4(x)$.

a.) By looking at the graph that you generated above using MATLAB and using `zoom on`, esti-
mate the numerical error (the distance) between the points $(\pi, \cos(\pi))$ and $(\pi, P_4(\pi))$?
(5) Answer: _____

Exercise 3:

a.) Use MATLAB to calculate the numerical error if you tried to approximate $\cos(\pi)$ by $p_4(\pi)$?
(6) Answer: _____

b.) Taylor's theorem says that the error $|R_4(\pi)|$ must be less than or equal to $|f^{(5)}(z)|/5!(\pi)^5$ for
some value of z between 0 and π. What is the largest this bound on the error could be?
(7) Answer: _____

Exercise 4:
Using the commands as above, generate the 7th Taylor polynomial for the function $f = \tan x$ and
answer the questions below:

a.) Look at the graph of the functions $\tan(x)$ and $P_7(x)$.
How good an approximation is P_7 for $\tan(x)$ on the interval $[-\pi, \pi]$?
(8) Circle all that apply:
1. the approximation is poor when x is near $\pi/2$
2. the approximation is good when x is near zero
3. the approximation is good when x is near $\pi/2$
4. the approximation is poor when x is near zero

b.) Using MATLAB, find $P_7(\pi/4)$
(9) Answer: _____

c.) Find $\tan(\pi/4)$
   (10) Answer: _____

d.) Using MATLAB, find $P_7(\pi/6)$
   (11) Answer: _____

e.) Find $\tan(\pi/6)$
   (12) Answer: _____

f.) The errors between $\tan x$ and $P_7(x)$ will be calculated at $\pi/4$ and $\pi/6$:

   - Compute the error between $\tan(\pi/4)$ and its approximation, $P_7(\pi/4)$.
     (13) Answer: _____

   - Compute the error between $\tan(\pi/6)$ and its approximation, $P_7(\pi/6)$.
     (14) Answer: _____

   - With respect to the magnitude of the errors, is this what you would have expected?
     (15) Circle one:
     1. no, the errors should be the same **2.** no, the error should be greater near $\pi/6$ **3.** none of the above **4.** yes, the error is smaller near $c = 0$

Exercise 5:
We will use MATLAB to approximate the function $f(x) = 2x^2 + \cos(3x)$.

   - First, make a graph of this function on the interval $[-1, 1]$.

   - Then use the **taylor** command to compute p3(x) and p5(x) for $f(x) = 2x^2 + \cos(3x)$.

   - Plot these functions making sure to *use the "hold on" and "prism" commands.* Notice that as you get far away from $c = 0$, the approximations are not as good.

   - Label each approximation using the gtext command:
             >> gtext('whatever you want as your label')
     For example, to label the 3th approximation, type:
             >> gtext('p3')
     Then place your mouse pointer at the point in the graph window where you want this label to appear and click on the mouse. The label will be printed in the location it appears in the graphing window.
     (Alternately, you can use the graphical interface provided in the figure window.)

Consider the following: How many terms do you need to have in the Taylor approximation to f in order to get a polynomial which agrees with f pretty closely on the entire interval $[-1, 1]$? Let's say that we want to find a Taylor approximation which has an error of at most 0.1 from the function f. To find out how many terms are needed, plot successively better and better approximations. For example, you might compute `p6`, `p6`, `p7` or even better approximations. Hint: Look at the endpoints.

You will soon find an approximation that is within 0.1 of the function f. Use the `zoom` command to tell how good your current approximation is, to determine if you need to make a better approximation.

a.) How many terms of the Taylor series did you have to include to find a Taylor approximation which was within, say, an error of 0.1 from the function f?
   (16) Answer:

b.) Submit the graphing window when it contains `f(x)` and `p5(x)` and the Taylor approximation that is within 0.1 of the function f. Label the graphs. (17) **Attach your graph to the worksheet.**