MTH233

Functions of Several Variables

Project 3– Exercises

NAME: ______________________________
SECTION: ______________________________
INSTRUCTOR: ______________________________

Exercise 1:
IMPORTANT: Let f(x,y) = $(x^3 - y^3)e^{-x^2-y^2}$ be the function used for this entire exercise.
Answer each of the following:

1. Graph a 3-dimensional view of this function. Find a good view that shows maximum, minimum and /or saddle points. (*Hint: the interesting part of the function is near the origin. So take your x and y values to be no larger than ±3 or so. Otherwise you will miss the fine detail at the origin.)*
   (1) **Attach your graph to the worksheet.**

2. What is the domain of this function?
   (2) Circle one:
   1. all $x \in R$ and $y > 0$
   2. all points (x, y) such that $x, y \in R$
   3. all $y \in R$ and $x > 0$
   4. $-3 < x < 3$ and $-3 < y < 3$

3. Determine partial derivatives:
   Find:

   a) f_x =
   (3) Circle one:
   1. `(x^3+y^3)*exp(-x^2-y^2)+2*exp(-x^2-y^2)*x^2`
   2. `-2*x*exp(-x^2-y^2)*(x^3+y^3)+3*exp(-x^2-y^2)*x^2`
   3. `3*exp(-x^2-y^2)*y^2-2*x*(y^3+x^3)*exp(-x^2-y^2)`
   4. `3*x^2*exp(-x^2-y^2)-2*(x^3-y^3)*x*exp(-x^2-y^2)`

b) $f_y =$

(4) Circle one:

1. `3*exp(-x^2-y^2)*x^2-2*x*x*exp(-x^2-y^2)*(x^3+y^3)`
2. `-3*y^2*exp(-x^2-y^2)-2*(x^3-y^3)*y*exp(-x^2-y^2)`
3. `-2*y*exp(-x^2-y^2)*(x^3+y^3)+3*exp(-x^2-y^2)*y^2`
4. `3*y^2-2*y*exp(x^2-y^2)`

c) $f_{xx} =$

(5) Circle one:

1. `-2*(x^3+y^3)*exp(-x^2-y^2)+4*x^2*exp(-x^2-y^2)*(x^3+y^3)`
`-12*x^3*exp(-x^2-y^2)+6*x*exp(-x^2-y^2)`
2. `6*x*exp(-x^2-y^2)*(-2)*x+12*x^3*exp(-x^2-y^2)`
3. `6*x*exp(-x^2-y^2)-12*x^3*exp(-x^2-y^2)-2*(x^3-y^3)*exp(-x^2-y^2)`
`+4*(x^3-y^3)*x^2*exp(-x^2-y^2)`
4. `4*x*y*exp(-x^2-y^2)-6*x*exp(-x^2-y^2)x^2`
`-6*exp(-x^2-y^2)*(x^3+y^3)*x^2`

d) $f_{xy} =$

(6) Circle one:

1. `-6*x^2*y*exp(-x^2-y^2)+6*y^2*x*exp(-x^2-y^2)`
`+4*(x^3-y^3)*x*y*exp(-x^2-y^2)`
2. `4*x*y*exp(-x^2-y^2)*(x^3+y^3)-6*x*exp(-x^2-y^2)*y^2`
`-6*y*exp(-x^2-y^2)*x^2`
3. `-2*(x^3+y^3)*exp(-x^2-y^2)+4*x^2*exp(-x^2-y^2)*(x^3+y^3)`
`+6*y*exp(x-^2-y^2)`
4. `12*x*y^3*exp(-x^2-y^2)*(x^3+y^3)+6*exp(-x^2-y^2)*x`

e) $f_{yy} =$

(7) Circle one:

1. `12*y^3*exp(-x^2-y^2)*(x^3+y^3)+6*exp(-x^2-y^2)*y`
`-12*y^3*exp(-x^2-y^2)`
2. `4*x*exp(-x^2-y^2)*(x^3+y^3)-6*y*exp(-x^2-y^2)`
`-6*x^2*exp(-x^2-y^2)`
3. `-6*y*exp(-x^2-y^2)+12*y^3*exp(-x^2-y^2)-2*(x^3-y^3)*exp(-x^2-y^2)`
`+4*(x^3-y^3)*y^2*exp(-x^2-y^2)`
4. `-2*(x^3+y^3)*exp(-x^2-y^2)+4*y^2*exp(-x^2-y^2)*(x^3+y^3)`
`-12*y^3*exp(-x^2-y^2)+6*y*exp(-x^2-y^2)`

4. Graph the contours (level curves) for this function (use n = 10).
   (8) **Attach your graph to the worksheet.**

5a) The gradient of $f(x,y)$, denoted: $\nabla f(x,y)$ is
   (9) Circle one:
   1.

$$\big(3x^2e^{-x^2-y^2} - 2x(x^3-y^3)e^{-x^2-y^2}\big)i + \big(-3y^2e^{-x^2-y^2} - 2y(x^3-y^3)e^{-x^2-y^2}\big)j$$

   2.

$$\big(2ye^{-x^2-y^2}(x^3+y^3) + 3e^{-x^2-y^2}y^2\big)i - \big(2xe^{-x^2-y^2}(x^3+y^3) + 3e^{-x^2-y^2}x^2\big)j$$

   3.

$$\big(-2(x^3+y^3)e^{-x^2-y^2} + 4x^2e^{-x^2-y^2}(x^3+y^3)\big)i - \big(2(x^3+y^3)e^{-x^2-y^2} + 4y^2e^{-x^2-y^2}(x^3+y^3)\big)j$$

5b) If the contours (or level curves) are far apart, is the $\|\nabla f\|$ large or small? Why?
   (10) Circle one:
   1. The $\|\nabla f\|$ is small because it is always proportional to the magnitude of $f_{xx}f_{yy} - f_{xy}^2$
   2. $\|\nabla f\|$ is large, because a large $\|\nabla f\| \rightarrow f_x$ and f_y are large, implying less contour lines.
   3. $\|\nabla f\|$ is small, because a small $\|\nabla f\| \rightarrow f_x$ and f_y are small, implying less contour lines.
   4. There is no relationship between the gradient and contour lines.

5c) If you were on the surface at a point (x,y), in what direction would you move to increase your altitude as fast as possible? Why?
   (11) Circle one:
   1. Opposite to the direction of the gradient, as it points in the direction of greatest ascent.
   2. In a direction perpendicular to the direction of the gradient, since it lies in the xy plane.
   3. In the direction of the gradient, since it points in the direction of greatest descent.
   4. In the direction of the gradient, as it points in the direction of greatest ascent.

6a) At which points (x, y) does f(x,y) have critical points?

(12) Circle one:

1.

$$(0,0),\ \left(0,\frac{\sqrt{6}}{2}\right),\ \left(0,-\frac{\sqrt{6}}{2}\right),\ \left(\frac{\sqrt{6}}{2},0\right),\ \left(-\frac{\sqrt{6}}{2},0\right)$$

2.

$$(0,0),\ \left(0,\frac{\sqrt{6}}{2}\right),\ \left(0,-\frac{\sqrt{6}}{2}\right),\ \left(\frac{\sqrt{6}}{2},0\right),\ \left(-\frac{\sqrt{6}}{2},0\right),\ \left(-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right),\ \left(\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}\right)$$

3.

$$(0,0),\ \left(0,\frac{\sqrt{6}}{2}\right),\ \left(0,-\frac{\sqrt{6}}{2}\right),\ \left(\frac{\sqrt{6}}{2},0\right),\ \left(-\frac{\sqrt{6}}{2},0\right),\ \left(\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right),\ \left(-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}\right)$$

6b) At which points (x, y, z) do the relative extrema occur? (that is, find the matching z value for each critical point):

(13) Circle one:

1.

$$(0,0,0)\ ,\ \left(0,\frac{\sqrt{6}}{2},-\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(0,-\frac{\sqrt{6}}{2},\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(\frac{\sqrt{6}}{2},0,\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(-\frac{\sqrt{6}}{2},0,-\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

$$\left(-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2},-\frac{3\sqrt{3}}{4e^{3/2}}\right)\ ,\ \left(\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2},\frac{3\sqrt{3}}{4e^{3/2}}\right)$$

2.

$$(0,0,0)\ ,\ \left(0,\frac{\sqrt{6}}{2},\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(0,-\frac{\sqrt{6}}{2},-\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(\frac{\sqrt{6}}{2},0,\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(-\frac{\sqrt{6}}{2},0,-\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

$$\left(\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2},\frac{3\sqrt{3}}{4e^{3/2}}\right)\ ,\ \left(-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2},-\frac{3\sqrt{3}}{4e^{3/2}}\right)$$

3. not listed

7a. Use the Second Partials Test to determine which critical points yield relative maxima, relative minima or saddle points, if any. What are the MATLAB commands that you used?

- First, what command defines d: (assume that the function $f$and all partial derivatives where defined, i.e., `fx, fy, fxx, fxy and fyy`)
  (14) Answer:

- What command finds the critical numbers and puts these numbers in variables a and b?
  (15) Answer:

- What commands perform the second partials test on the first critical number (a, b)?
  (16) Answer:

7b) At which points (x, y, z) do the **relative maxima** occur?
(17) Circle one:
1.

$$\left(0, -\frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right), \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)$$

2.

$$\left(0, \frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right), \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

3.

$$\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right), \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)$$

4. not listed

7c) At which points (x, y, z) do the **relative minima** occur:

(18) Circle one:

1.

$$\left(0, -\frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right), \ \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)$$

2.

$$\left(0, \frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right), \ \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

3.

$$\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right), \ \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)$$

4. not listed

7d) At which points (x, y, z) do the **saddle points** occur:

(19) Circle one:

1.

$$\left(0, -\frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right), \ \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)$$

2.

$$\left(0, \frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right), \ \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

3.

$$\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right), \ \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)$$

4. not listed

8. Type <u>**hold on**</u> to hold the contour graph and use the **gradient** and **quiver** commands to draw a plot of the gradient vectors. Use the **text** command to label the local extrema and saddle points on this plot. Label any local maximum 'max ', local minimum 'min' and saddle points 'sdl '.
(20) **Attach your graph to the worksheet.**

8a) Around a local minimum, in what directions do the gradient vectors point? Why?
(21) Circle one:
1. The vectors point away from the minimum, because the gradient points in the direction of most rapid ascent.
2. The vectors point in towards the minimum, because the gradient points in the direction of most rapid descent.
3. The vectors point every which way, since there is no relationship between the gradient and extrema.

8b) Around a local maximum, in what directions do the gradient vectors point? Why?
(22) Circle one:
1. The vectors point away from the maximum, because the gradient points in the direction of most rapid ascent.
2. The vectors point in towards the maximum, because the gradient points in the direction of most rapid ascent.
3. The vectors point every which way, since there is no relationship between the gradient and extrema.

8c) Around a saddle point, what happens to the directions of the gradient vectors? Why?
(23) Circle one:
1. The vectors point every which way, since there is no relationship between the gradient and extrema
2. The vectors point away from saddle point, because the gradient points in the direction of most rapid descent.
3. The vectors point both toward and away from the saddle point, as it is both a maximum and a minimum.