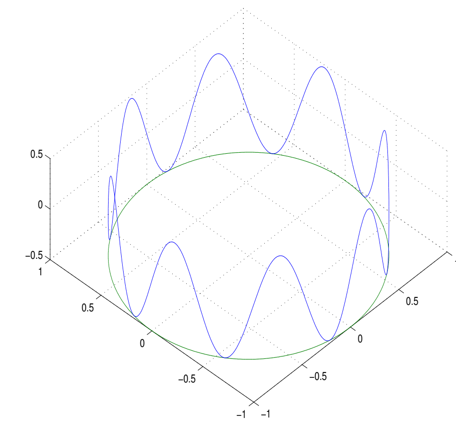
The College of Staten Island

Department of Mathematics



MTH 233

Calculus III

http://www.math.csi.cuny.edu/matlab/

MATLAB PROJECTS

STUDENT: ______________________________

SECTION: ______________________________

INSTRUCTOR: __________________________

BASIC FUNCTIONS

Elementary Mathematical functions		
MATLAB notation	Mathematical notation	Meaning of the operation
`sqrt(x)`	$\sqrt{x}$	square root
`abs(x)`	$\|x\|$	absolute value
`sign(x)`		sign of x ($+1$, -1, or 0)
`exp(x)`	e^x	exponential function
`log(x)`	$\ln x$	natural logarithm
`log10(x)`	$\log_{10} x$	logarithm base 10
`sin(x)`	$\sin x$	sine
`cos(x)`	$\cos x$	cosine
`tan(x)`	$\tan x$	tangent
`asin(x)`	$\sin^{-1} x$	inverse sine
`acos(x)`	$\cos^{-1} x$	inverse cosine
`atan(x)`	$\tan^{-1} x$	inverse tangent

MTH233

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3-Dimensional Graphs

NEW MATLAB COMMANDS:

plot3(x,y,z)
zlabel('   ')
text(a,b,c,'   ')
view(angle,elevation)
dot(u,v)
cross(u,v)
meshgrid(a:h:b)
surf(x,y,z)
mesh(x,y,z)
view([a,b,c])

1 Objective

In calculus III, we are interested in 3-dimensional problems. MATLAB has a command **plot3** which plays a very similar role as `plot(x,y)`. MATLAB also has other commands – **surf** and **mesh** which we shall need in order to graph different kinds of 3-dimensional figures. When you complete this project, you should be able to draw and interpret 3-dimensional graphs using MATLAB.

2 3-Dimensional Vectors

2.1 Plotting 3-D Vectors

We begin by plotting directed line segments in space. In general, a vector connecting two points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ has an equivalent or positional vector $\overrightarrow{PQ} = \langle x_2 - x_1\ ,\ y_2 - y_1\ ,\ z_2 - z_1\rangle$ such that its initial point or "tail" is at the origin, $(0, 0, 0)$.

Example 1:
Plot the vector determined by the two points $P = (2, 1, 3)$ and Q=$(-1, 2, 1)$.

```
>> plot3([2 -1],[1 2],[3 1],'r'),grid  % connect P and Q with a red line
```

Now plot an equivalent blue vector originating at $(0, 0, 0)$. If we are going to be plotting many equivalent vectors using large numbers, the following will save us some arithmetic:

```
>> p=[2 1 3];q=[-1 2 1];
                              % p(1) =2; p(2)=1;p(3)=3; q(1)=-1;q(2)=2;q(3)=1
>> hold on, plot3([0  q(1)-p(1)],[0  q(2)-p(2)],[0 q(3)-p(3)],'b')
```

These commands work for any points P and Q you wish to define.
Additionally, to label the points....

```
>> xlabel('x'),ylabel('y'),zlabel('z') % label the x,y and z axes
>> text(2,1,3,'(2,1,3)')  %label the points
>> text(-1,2,1,'(-1,2,1)')
>> hold on
```

These commands help you keep track of what is going on in each of the three dimensions. Finally, there is an `axis` command for 3-D graphs. This command will zoom out on the two vectors:

```
>>  axis([-4 4 0 8 -8 5])
```

Like its 2-dimensional counterpart, the command axis([xmin xmax ymin ymax zmin zmax]) changes the bounds of the graph.

Exercise 1:
Use MATLAB to plot 2 vectors, a blue vector connecting the points $P = (1, -3, 5)$ and $Q = (3, 2, 6)$ and an equivalent red vector which has as its initial point the origin, (0,0,0).

a.) What commands define the points p and q?
(1) Circle one:
1. `p=[3 2 6];q=[1 -3 5];`
2. `p=[1 -3 5];q=[3 2 6];`
3. `p=[5 -3 1];q=[3 2 6];`
4. not listed

b.) One or more of the following commands will plot the blue vector. Which ones are they?
(2) Circle all that apply:
1. `plot3([1 3],[-3 2],[5 6],'b'),grid`
2. `plot3([p(1) q(1)],[p(2) q(2)],[p(3) q(3)],'b'),grid`
3. `plot3([0 3],[0 2],[0 6],'b'),grid`

c.) What command plots the equivalent red vector?

(3) Circle one:

1. `plot3([p(1) q(1)],[p(2) q(2)],[p(3) q(3)]),grid`
2. `hold on, plot3([0  q(1)-p(1)],[0  q(2)-p(2)],[0 q(3)-p(3)],'r')`
3. `hold on, plot([0  q(1)-p(1)],[0  q(2)-p(2)],[0 q(3)-p(3)])`
4. not listed

d.) What does it mean for two vectors to be equivalent? *(Note: "same direction" would necessarily imply being parallel, but the converse is not necessarily so.)*

(4) Circle one:

1. if they are parallel
2. their magnitudes are the same
3. their directions are the same
4. their magnitude and direction are the same

It is possible for unparallel 3-D vectors, depending on the perspective or viewpoint that one has, to falsely appear to be parallel. We explore this in the next example. There we introduce a new command, `view( )`.

◇ `view(angle,elevation)` **sets the viewing point for a 3-dimensional plot.**

angle is the horizontal rotation about the z-axis (in degrees). **elevation** is the vertical height (in degrees). If the elevation is positive, you are viewing the object from above the xy-plane. If the elevation is negative, you are viewing the object from below. The default 3-dimensional view is angle=-37.5 and elevation=30.

Example 2:
Consider these 2 vectors, one connects the points $P = (4, -1, 3)$ and $Q = (7, 2, 8)$, and the other is a vector $\vec{v} = \langle 1, 4, 5 \rangle$ having its initial point at the origin.

```
>> plot3([4 7],[-1 2],[3 8],'r'),grid          % through p and q
>> hold on, plot3([0  1],[0  4],[0 5],'b')   % inequivalent vector
```

We can see that these vectors are very inequivalent – they point in different directions *and* are of differing magnitudes. But suppose curiosity got the better of you, and you decided to change your viewpoint with the following command:

```
>> view(83.5,  8)
```

The command `view(83.5, 8)` places the viewpoint $83.5°$ in a counterclockwise direction from the negative y-axis and the elevation of the viewpoint is $8°$ above the xy-plane.
Experimenting with various viewpoints may be either illuminating *or* misleading. In this case, it

could have led us to the false assumption that the vectors where actually parallel. We can conclude that perspective in space is not as straightforward as in the xy-plane.

2.1.1 The Rotate Tool

You can try different views of your own using the `view` command. Alternately, new with MATLAB Release 11 is a rotate tool located atop the figure window. It is activated by clicking on the button labeled with a circular arrow. You then use the mouse in a "drag and drop" fashion to rotate the image. Note that as you hold the left clicker down, and drag the mouse, that the azimuth and elevation are displayed in the bottom left-land corner of the figure window. When you release the left clicker, the image reappears displaying the new viewpoint. If you want to try several viewpoints quickly, this is the way to go.

2.2 The Dot and Cross Vector Operations

MATLAB can help us do vector calculations.

Example 3:
In order to calculate the dot product of two vectors, $\vec{u} = \langle 2, -4, -1\rangle$ and $\vec{v} = \langle 3, 1, -2\rangle$, the MATLAB command is $\mathbf{dot(u,v)}$. For example,

```
>> u=[2 -4  -1];v=[3  1  -2];
>> d=dot(u,v)
```

MATLAB computes d=4. Observe that the dot product is a scalar.
In order to calculate the cross product of two vectors, $\vec{u}$ and $\vec{v}$, the MATLAB command is $\mathbf{cross(u,v)}$. For example....

```
>> c=cross(u,v)
```

MATLAB computes c to be 9 1 14. This is MATLAB's way of writing the vector $9\vec{i} + \vec{j} + 14\vec{k}$.

Exercise 2:

a.) Compute dot(u,c) and dot(v,c) where u = [3 -5 1], v = [3 2 -2] and c = [8 9 21]

- dot(u,c)=
  (5) Answer:

- dot(v,c)=
  (6) Answer:

- What do your answers to `dot(u,c)` and `dot(v,c)` indicate?
  (7) Circle all that apply:
  1. u and v are perpendicular
  2. $u \times v = c \rightarrow\ u \bullet c = v \bullet c = 0$
  3. $u \bullet c = v \bullet c = 0 \rightarrow$ that u and v are mutually $\perp$ to c
  4. $u \bullet c = v \bullet c = 0$ is mere coincidence and has no implications

b.) Plot u in green, v in blue and c in red. Using view or the rotate tool, find a viewpoint which illustrates the relationship between $\vec{u}$,$\vec{v}$ and $\vec{c}$. Submit the graph.
(8) **Attach your graph to the worksheet.**

3 Graphing Surfaces

Whereas commands like `plot3`, `cross` and `dot` are used with vectors, another group of commands are useful for plotting planes and other 3-D surfaces.

Example 4:

We want to graph the equation of the plane containing the vectors $\vec{u} = \langle 2, -4, -1\rangle$ and $\vec{v} = \langle 3, 1, -2\rangle$. Recall that $u \times v = \langle 9, 1, 14\rangle$. This plane is determined by $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ where $\langle a, b, c\rangle$ is a vector normal to the plane and $\langle x_1, y_1, z_1\rangle$ is any point in the plane. Solving for z, $9(x - 0) + 1(y - 0) + 14(z - 0) = 0$ becomes $z = -(9x + y)/14$. So to graph the plane....

```
>> [x,y]=meshgrid(-2:1/4:2);
>> z=-(9*x+y)/14;
>> surf(x,y,z)
```

You might want to again use the rotate tool to see the plane clearer.
How does meshgrid work?
meshgrid(0:0.1:2) forms a 2-dimensional grid in the xy-plane by starting on the x-axis at 0 and marking points on the x-axis at .1 increments until it reaches 2 and repeating the same process along the y-axis. To complete the grid, lines parallel to the y-axis are drawn through the points $(x_i, 0)$ and $(x_i, 2)$ for each x_i in the array of x-values and lines parallel to the x-axis are drawn

through the points $(0, y_i)$ and $(2, y_i)$ for each y_i in the array of y-values. MATLAB has created a meshgrid with points like (.1,1.5) and (.1,1.6) and (.2,1.5) and (.2,1.6) forming the vertices of a "mesh" mini-square.

surf(x,y,z) plots the surface determined by the z-values for each point in the xy mesh grid. For example, if $z = f(x, y)$ and the meshgrid has points like (.1,1.5) and (.1,1.6) and (.2,1.5) and (.2,1.6) forming the vertices of a "mesh" mini-square, then MATLAB computes $f(.1, 1.5)$ and $f(.1, 1.6)$ and $f(.2, 1.5)$ and $f(.1, 1.6)$. MATLAB then connects, for example, the points $\big((.1,1.5), f(.1,1.5)\big)$ to $\big((.1,1.6), f(.1,1.6)\big)$ with a line. The underlying square grid induces 4-sided patches on the surface. MATLAB will not connect $\big((.1,1.5), f(.1,1.5)\big)$ and $\big((.2,1.6), f(.2,1.6)\big)$ with a line since in the meshgrid in the xy-plane, (.1,1.5) and (.2,1.6) are not connected by a line.

Exercise 3:
Use MATLAB to graph the plane through the origin which contains the vectors $\vec{u} = \langle 6, 4, -1\rangle$ and $\vec{v} = \langle -3, 12, 5\rangle$. On the same graph, plot the cross product of these vectors. Experiment with the **view** and **axis** commands to obtain the best viewpoint that you can. *(Hint: It might be easier to graph the cross product if you can find a shorter vector in the same direction.)*

a.) What is your equation for the plane? $z =$
  (9) Answer:

b.) Submit the graph of the plane and cross product on one graph.
  (10) **Attach your graph to the worksheet.**

Example 5:
Plot three views of the equation $y^2 + z^2 = 4$ from the points a. (10,0,0) b.(0,10,0) c. (10,10,10)
MATLAB COMMANDS

```
>> [x,y]=meshgrid(-2:.25:2);
>> z=sqrt(4-y.^2);
>> mesh(x,y,z)
>> xlabel('x');ylabel('y');zlabel('z');
>> hold on
>> z1=-sqrt(4-y.^2);
>> mesh(x,y,z1)
>> view([10 0 0])
>> view([0 10 0])
>> view([10 10 10])
```

EXPLANATION: mesh(x,y,z) is a 3-dimensional graphing command which operates in a manner very similar to **surf(x,y,z)**. The command **view([a b c])** shows the graph as viewed from the direction of the point (a,b,c). Thus, the magnitude of (a,b,c) is ignored, and the commands **view([10 0 0])** and **view([1 0 0])** produce the same graph.

Exercise 4:
Use MATLAB to graph the following surfaces. Use **subplot** and print all of these graphs on one sheet! Label the graphs. Change the view on each graph to highlight the behavior of the graph near the origin. Based on your graphs, try to determine the value of the function when x=0 and y=0 – if it exists.

a.) Plot the following functions, and compute $f(0,0)$ if it exists:

1.) $z_1 = x^2 + y^2$
At $x = 0$, $y = 0$, $z_1 = ?$
(11) Answer:

2.) $z_2 = e^{x^2+y^2}$
At $x = 0$, $y = 0$, $z_2 = ?$
(12) Answer:

3.) $z_3 = 1/(x^2 + y^2)$
At $x = 0$, $y = 0$, $z_3 = ?$
(13) Answer:

4.) $z_4 = \sin(x^2 + y^2)/(x^2 + y^2)$
At $x = 0$, $y = 0$, $z_4 = ?$
(14) Answer:

b.) Submit the graph.
(15) **Attach your graph to the worksheet.**

Recall that the **subplot** command divides the graphing window into separate plotting areas and allows you to put many graphs on one page. The command **subplot(3,3,1)** will be the upper left most plot in a three row by three column plot.

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3-Dimensional Graphs – Vector Valued Functions

1 Drawing Vector Valued Functions

A vector valued function is simply a function $r = \langle x(t), y(t), z(t) \rangle$ for which each value of t gives a vector back as its value. We can think of the function as describing the position of a spaceship as it flies around. The parameter t can be thought of as time. MATLAB has powerful capabilities to draw such functions.

Let's illustrate those capabilities now, using the vector valued function $r = \langle x(t), y(t), z(t) \rangle$ where $x(t) = \sin(t),\ y(t) = \cos(t),\ and\ z(t) = t$ on the domain $0 \le t \le 4\pi$. One way to graph r is to use the **comet3** command, as below:

```
>> t=linspace(0,4*pi,500);    % set up the domain using 500 points
>> comet3(sin(t),cos(t),t)
```

This command graphs the curve dynamically in your figure window. Although the **comet3** command produces snazzy-looking output, a more permanent way to graph the curve is with the **plot3** command, which plots the curve all at once.

```
>> plot3(sin(t),cos(t),t)
```

When we expect to refer to the values of the curve r later on, we can put them into a MATLAB variable for safekeeping. For example, the command

```
>> r=[sin(t); cos(t); t]
```

will create an array whose first row contains the x coordinates of $r(t)$ as t progresses, whose second row has the y coordinates, and whose last row has the z coordinates. After making r, you can get the x coordinates with r(1,:) and the y coordinates with r(2,:), etc. But sometimes it is easier to plot a function $r = \langle x(t), y(t), z(t) \rangle$ by first defining x, y and z separately.

Example 1:
Draw the curve given by the function $r(t) =< \cos t, \sin t, \cos 4t \sin 4t >$, on the domain $0 \leq t \leq 2\pi$.
Solution: First we set up the function.

```
>> t=linspace(0,2*pi,500);
>> x=cos(t);
>> y=sin(t);
>> z=cos(4*t).*sin(4*t);
```

Next, we plot the function.

```
>> comet3(x,y,z)          % this gives a comet plot.
>> plot3(x,y,z)           % this gives a permanent plot.
```

Note: graphs to be submitted will only work if generated with plot3.
Let's conclude this section by drawing some nice-looking vector valued functions.

Exercise 1:
Make a graph of the vector valued function:

$$r(t) = \cos^2(2t)\vec{i} + \sin^2(3t)\vec{j} + \cos(2t - \pi/2)\vec{k}$$

a.) What are the MATLAB commands you used to generate the graph?
(1) Answer:

b.) What is the period of this parametric equation?
(*Hint: Use comet3 and different domains for t.*)
(2) Circle one:
1. *The function has no period*
2. *the period is* π
3. *the period is* 2π
4. *the period is* 4π

c.) Submit your graph.
(3) **Attach your graph to the worksheet.**

2 Representing The Velocity and Acceleration Vectors

It is easy to draw vector valued functions with MATLAB, as we saw in the previous section. In this section, we will learn how to draw the velocity vector at a point on the curve. Let's go back to the function $r(t) = \langle \sin t, \cos t, t \rangle$, which we discussed in the first section. This function describes the path of curve which spirals upwards around the z-axis. In class, we learned that the velocity vector $v(t) = \langle \cos t, -\sin t, 1 \rangle$ – which is obtained by simply differentiating the function r in each component – describes the direction of the curve.
The **acceleration** vector $a(t) = \langle -\sin t, -\cos t, 0 \rangle$ is similarly obtained by differentiating the velocity v in each component. Our goal is to draw the curve r, and indicate at a few points how the velocity and acceleration vectors relate to the curve itself. Below we will use the **ones** and **zeros** commands to give us a whole row of ones or zeros, as we need.

```
>> t=linspace(0,4*pi,500);                % use 500 points for t
>> r=[ sin(t); cos(t); t];                % the curve itself
>> v=[cos(t); -sin(t); ones(1,500)];      % the velocity vector - the third row is 500 1's.
>> a=[-sin(t); -cos(t); zeros(1,500)];    % the acceleration vector - the third row is 500 0's.
>> plot3(r(1,:),r(2,:),r(3,:))            % draw the curve
>> hold on                                % preserve the plot
```

Now let's pick some random point, say the 144^{th} point in t, and illustrate how the velocity vector is tangent to the curve. To draw a line from the point $\langle x0, y0, z0 \rangle$ to the point $\langle x, y, z \rangle$, you use the command **plot3(x0+[0 x],y0+[0 y],z0+[0 z])**.

```
>> n=144; % we will use the 144th point
>> x0=r(1,n);y0=r(2,n);z0=r(3,n); % the special point
>> plot3(x0,y0,z0,'*r') % put a red star at the point
>> plot3(x0+[0 v(1,n)],y0+[0 v(2,n)],z0+[0 v(3,n)],'g') % draw v in green
>> plot3(x0+[0 a(1,n)],y0+[0 a(2,n)],z0+[0 a(3,n)],'m') % draw a in magenta
```

Exercise 2:
The approach to the Lincoln Tunnel in New York City from New Jersey resembles a helix. A possible model for this road is given by the parametric equations $r(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$ where

$$x(t) = (1-t)\cos 4\pi t$$

$$y(t) = (1-t)\sin 4\pi t$$

$$z(t) = 1-t$$

$$and\ 0 \leq t \leq 1$$

a.) Compute the velocity vector $v(t)$ and the acceleration vector $a(t)$. Then generate these vectors using 500 points for t. (You can find v and a symbolically using `diff`, being careful not to reassign any variables.)

- $v(t)$ is
  (4) Circle one:
  1. $\big(-\sin(4\pi t) + 4\pi(1-t)\cos(4\pi t)\big)i \; + \; \big(-\cos(4\pi t) - 4\pi(1-t)\sin(4\pi t)\big)j - k$
  2. $\big(-\cos(4\pi t) - 4\pi(1-t)\sin(4\pi t)\big)i \; + \; \big(4\pi(1-t)\cos(4\pi t) - \sin(4\pi t)\big)j - k$
  3. $\big(-\cos(4\pi t) + 4\pi(1-t)\sin(4\pi t)\big)i \; + \; \big(-\cos(4\pi t) - 4\pi(1-t)\sin(4\pi t)\big)j - k$
  4. none of the above

- $a(t)$ is
  (5) Circle one:
  1. $\big(8\pi\sin(4\pi t) - 16\pi^2(1-t)\cos(4\pi t)\big)i \; + \; \big(-8\pi\cos(4\pi t) - 16\pi^2(1-t)\sin(4\pi t)\big)j$
  2. $\big(8\pi\cos(4\pi t) + 16\pi^2(1-t)\cos(4\pi t)\big)i \; - \; \big(8\pi\sin(4\pi t) - 16\pi(1-t)\cos(4\pi t)\big)j$
  3. $\big(-8\pi\cos(4\pi t) - 4\pi(1-t)\cos(4\pi t)\big)i \; + \; \big(8\pi\sin(4\pi t) - 4\pi(1-t)\sin(4\pi t)\big)j$
  4. none of the above

b.) Generate the graph for $r(t)$. Plot the velocity and acceleration vectors when $t = 1/2$ on this graph. Label the velocity and acceleration vectors. Submit the graph of $r(t)$ with the indicated velocity and acceleration vectors.
(6) **Attach your graph to the worksheet.**

3 Animated Graphing and `csimovie.m`

As has been seen, plotting $\vec{v}$ and $\vec{a}$ for even a few values of t can be time consuming. We will now view the curve described by $r(t)$ on $[t_1, t_n]$ along with $\vec{v}(t_i)$ and $\vec{a}(t_i)$ (where $i = 1, 2, \ldots, n$) in n time frames – an animation of the behavior of $\vec{v}$ and $\vec{a}$ in space *over time*. To facilitate this, we have written our own function file, **csimovie**, which utilizes a built-in function called `movie`.
Type

```
>> help csimovie
```

at the command prompt. If help is not displayed, download `csimovie.m` from our website at "http://www.math.csi.cuny.edu/Courses/MTH229", right-click on "csimovie.m" and save it to the appropriate directory.

Example 2:
try the following:

```
>> syms t
>> M=csimovie(sin(t),cos(t),t,0,4*pi,25,30,40,1,1,1);
```
% use t's, not x's. Also, be sure to include a semicolon (;) or pages of data will be displayed.

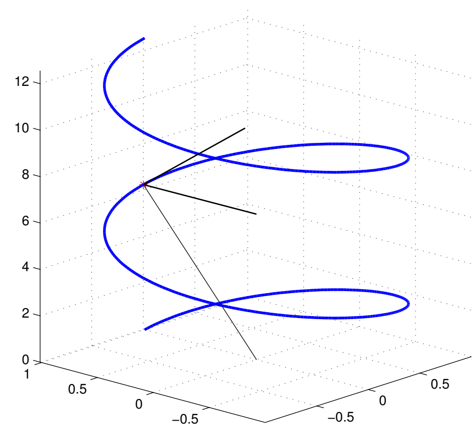


Figure 1:

$$r(t) = \sin t\vec{i} + \cos t\vec{j} + t\vec{k}, \quad v(2\pi) = \langle \cos 2\pi, -\sin 2\pi, 1\rangle, \quad a(2\pi) = \langle -\sin 2\pi, -\cos 2\pi, 0\rangle$$

format: (for reference, don't type this)
- M=csimovie(x(t),y(t),z(t),a,b,N,AZ,EL,size,vel,acc);

In the example, r is set up where $x(t)$=`sin(t)`, $y(t)$=`cos(t)` and $z(t)$=`t`. `a=0` and `b=4*pi` establish the time interval of $[0, 4\pi]$. The movie plays **N**=25 frames, $[t_1, t_2, \ldots, t_{25}]$ where $t_1 = 0, t_2 = \pi/6, \ldots, t_{25} = 4\pi$. First, $r(t)$ is traced out on $[0, 4\pi]$ along with $\vec{a}(t_1)$ and $\vec{v}(t_1)$. The graph is then wiped clean. It again plots, this time $r(t)$ is traced out on $[0, 4\pi]$ along with $\vec{a}(t_2)$ and $\vec{v}(t_2)$. Rapidly, this continues until t_{25}. Thus, 25 snapshot-like images of the function will be spliced together and viewed sequentially, creating an animated view not unlike the frames of a movie reel. Note that the position vector is plotted in red, velocity in green, and acceleration in violet.

Replay the movie
The animation data has been stored in a matrix variable **M**. Type **movie(M,-5)** and the movie

plays forward and backward five times. Various PC's will process at different speeds: To decrease its speed, type **movie(M,-5,6)** and it plays six frames per second instead of the default 12 frames/sec. **movie(M,5,8)** plays the movie five times, forward only, at 8 frames/sec.

When plotting different functions:
In some cases, a large magnitude v or a might prohibit a good view of $r(t)$. To compensate for this, use the scalars VEL and ACC: VEL$\cdot\vec{v}(t)$ and ACC$\cdot\vec{a}(t)$. To reduce $||v||$ and $||a||$, set these values somewhere between 0 and 1, say VEL=0.5 and ACC=0.2.
If the PC locks up: You can usually get out of trouble by pressing the **CTRL** button on the keyboard and the letter **c** at the same time. You might have to try different values for **a**,**b**, and **N** so that you're not generating too many data points.

Exercise 3:
Create a movie for $r(t) = \sqrt{4-t^2}\cos(2\pi t)\vec{i} + \sqrt{4-t^2}\sin(2\pi t)\vec{j} + t\vec{k}$
Be careful, what is the domain of this function?
Try values `N=100,AZ=45,EL=20,VEL=0.1,ACC=0.01` for csimovie parameters.

a.) Based on the movie, is $r'(t)$ perpendicular to $r(t)$?
(7) Circle one: **1.** yes **2.** no

b.) Does $r'(t) \bullet r(t) = 0$?
(8) Circle one: **1.** yes **2.** no

c.) Based on the movie, is the speed constant?
(9) Circle one: **1.** yes **2.** no

d.) Compute $\|r'(t)\|$. Does $\|r'(t)\|$ = a constant?
(10) Circle one: **1.** yes **2.** no

e.) $\|r(t)\|^2 =$
(11) Circle one:
1. 2 **2.** $8-t^2$ **3.** $(4-t^2)(\cos(2\pi t) + \sin(2\pi t)) + t^2$ **4.** 4 **5.** none of the above

f.) $r(t)$ lies on the surface of
(12) Circle one:
1. a cone
2. sphere of radius 2
3. paraboloid
4. sphere of radius 4
5. none of the above

MTH233

The College of Staten Island
Department of Mathematics

Functions of Several Variables

1 NEW MATLAB COMMANDS

1.1 Symbolic Commands

syms f x y z a b Use this command to define the "symbolic" variables, in this case, $f\ x\ y\ z\ a\ b$.

[a,b]=solve(f1,f2) solves the 2 symbolic equations $f1 = 0$ and $f2 = 0$ simultaneously.

subs(subs(f,x,a),y,b) Symbolic substitution of functions of two variables: Note that f is a symbolic function representing $f(x, y)$. MATLAB evaluates and displays $z = f(a, b)$ for whatever appropriate values for a and b that you choose.

fx=diff(f,x) –will compute the partial derivative of f with respect to x.

fy=diff(f,y)–will compute the partial derivative of f with respect to y.

Note that the use of single quotes has been eliminated for defining symbolic variables in MATLAB 5

1.2 Standard Commands

surfc(x,y,z)
meshc(x,y,z)
These two MATLAB commands are the same as **surf(x,y,z)** and **mesh(x,y,z)** except that a contour plot is drawn beneath the surface.

contour(x,y,z) draws a contour plot of the matrix z. The contours are level curves in the units of the array z. The number of contour curves and their values are chosen automatically by contour.

contour(x,y,z,n) draws a contour plot with n contour levels.

[dx,dy]=gradient(z); computes a numerical approximation to the gradient field of the function z. The result is ordinarily 2 matrices, dx and dy, which are the same size as z, and contain horizontal and vertical first differences. IMPORTANT: Make sure to type a semi-colon at the end of the gradient command so that you do not display the entire matrices.

quiver(x,y,dx,dy) draws arrows at every pair of elements in matrices x and y. The pairs of elements in matrices dx and dy determine the direction and relative magnitude of the arrows. To make the arrows larger by a factor of 3, you can simply write *quiver(x,y,dx,dy,3)*.

Example 1:
Our objective is to graph the contour plot of $f(x, y) = x^4 - x^2 + y^3 + y^2$, to graphically determine and label its minimums, maximums and saddle points. We then confirm our conjectures using the second partials test. You should consult your calculus text on the use of the second partials test and the gradient before continuing.

- Find the partial derivatives of $f(x, y) = x^4 - x^2 + y^3 + y^2$ with respect to x and to y:

  first define the necessary variables. It is only necessary to define x *and* y
  `>> syms x y`
  Define f. *Note that there are no dots "."*
  `>>f=x^4-x^2+y^3+y^2;`
  Then find the derivatives:
  `>> fx=diff(f,x)` % The partial of f with respect to x
  `>> fy=diff(f,y)` % The partial of f with respect to y
  MATLAB responds with:
  `fx = 4*x^3-2*x and fy = 3*y^2+2*y`

- Solve `fx=0` and `fy=0` simultaneously for the critical numbers:

  `>> [a,b]=solve(fx,fy)`
  MATLAB responds with

```
a =                          b =

[            0]               [    0]
[            0]               [ -2/3]
[  1/2*2^(1/2)]               [    0]
[ -1/2*2^(1/2)]               [    0]
[  1/2*2^(1/2)]               [ -2/3]
[ -1/2*2^(1/2)]               [ -2/3]
```

To access the first pair of (x, y) points, use:

```
>> a(1),b(1)
```
for the second pair...
```
>> a(2),b(2)
```
and so forth.

Thus, the solutions are:
$(0,0)$, $(0,-2/3)$, $(2^{1/2}/2,0)$, $(-2^{1/2}/2,0)$, $(2^{1/2}/2,-2/3)$, $(-2^{1/2}/2,-2/3)$
You use the critical numbers above in the second partials test:

- First, find the function used for the second partial. Consult your Calculus text for the formula:

```
>> fxx=diff(fx,x);
>> fyy=diff(fy,y);
>> fxy=diff(fx,y);
>> d=fxx*fyy-fxy^2
```

- Now substitute the critical numbers into d. "a(1)" is substituted into the x values, and "b(1)" is substituted into the y values. Do the same for "a(2), b(2)" through "a(6), b(6)":

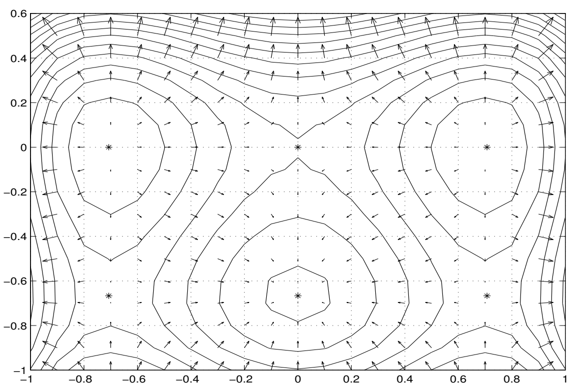
```
>> subs(subs(d,x,a(1)),y,b(1))
```

The remainder of the second partials test for determining extrema is left as an exercise. The **subs** command will be useful in determining its outcome. You will compare these results with the following contour graph:

- Above we made use of symbolic commands to algebraically determine extrema. In the remaining portion of the example, we restrict ourselves to standard MATLAB commands for plotting functions:
We wish to graph $f(x)$ and label its extrema, but we do not want to disturb our defined symbolic variables, namely f, x, y, and especially a and b – as these are our critical points! Thus, we will take care in naming our numeric (or non-symbolic) variables with different names:

```
>> [x1,y1]=meshgrid(-1:1/10:1,-1:1/10:.6); % −1 ≤ x ≤ 1, −1 ≤ y ≤ 0.6
>> z1=x1.^4-x1.^2+y1.^3+y1.^2;
>> [dx1,dy1]=gradient(z1);
```

```
>> contour(x1,y1,z1,12)
>> hold on
>> plot(single(a),single(b),'*r') % must convert symbolic a and b to
numeric for plotting
>> quiver(x1,y1,dx1,dy1)
>> grid
```



You should be observing in your figure window a contour graph with critical points in red along with the gradient vectors. To compare the level curve with the 3-d view:

```
>> figure(2)
>> meshc(x1,y1,z1)
```

It will be left as an exercise as to which points are minimums, which are maximums, and whether there are saddle points. Compare your assertions based on the graph with your second partial test results. Good luck!

Exercise 1:
IMPORTANT: Let f(x,y) = $(x^3 - y^3)e^{-x^2-y^2}$ be the function used for this entire exercise.
Answer each of the following:

1. Graph a 3-dimensional view of this function. Find a good view that shows maximum, minimum and /or saddle points. (*Hint: the interesting part of the function is near the origin. So take your x and y values to be no larger than* ± 3 *or so. Otherwise you will miss the fine detail at the origin.)*
   (1) **Attach your graph to the worksheet.**

2. What is the domain of this function?

(2) Circle one:
1. all $x \in R$ and $y > 0$
2. all points (x, y) such that $x, y \in R$
3. all $y \in R$ and $x > 0$
4. $-3 < x < 3$ and $-3 < y < 3$

3. Determine partial derivatives:
Find:

a) f_x =

(3) Circle one:
1. `(x^3+y^3)*exp(-x^2-y^2)+2*exp(-x^2-y^2)*x^2`
2. `-2*x*exp(-x^2-y^2)*(x^3+y^3)+3*exp(-x^2-y^2)*x^2`
3. `3*exp(-x^2-y^2)*y^2-2*x*(y^3+x^3)*exp(-x^2-y^2)`
4. `3*x^2*exp(-x^2-y^2)-2*(x^3-y^3)*x*exp(-x^2-y^2)`

b) f_y =

(4) Circle one:
1. `3*exp(-x^2-y^2)*x^2-2*x*x*exp(-x^2-y^2)*(x^3+y^3)`
2. `-3*y^2*exp(-x^2-y^2)-2*(x^3-y^3)*y*exp(-x^2-y^2)`
3. `-2*y*exp(-x^2-y^2)*(x^3+y^3)+3*exp(-x^2-y^2)*y^2`
4. `3*y^2-2*y*exp(x^2-y^2)`

c) $f_{xx} =$

(5) Circle one:
1. `-2*(x^3+y^3)*exp(-x^2-y^2)+4*x^2*exp(-x^2-y^2)*(x^3+y^3)`
`-12*x^3*exp(-x^2-y^2)+6*x*exp(-x^2-y^2)`
2. `6*x*exp(-x^2-y^2)*(-2)*x+12*x^3*exp(-x^2-y^2)`
3. `6*x*exp(-x^2-y^2)-12*x^3*exp(-x^2-y^2)-2*(x^3-y^3)*exp(-x^2-y^2)`
`+4*(x^3-y^3)*x^2*exp(-x^2-y^2)`
4. `4*x*y*exp(-x^2-y^2)-6*x*exp(-x^2-y^2)x^2`
`-6*exp(-x^2-y^2)*(x^3+y^3)*x^2`

d) $f_{xy} =$

(6) Circle one:

1. `-6*x^2*y*exp(-x^2-y^2)+6*y^2*x*exp(-x^2-y^2)`
`+4*(x^3-y^3)*x*y*exp(-x^2-y^2)`

2. `4*x*y*exp(-x^2-y^2)*(x^3+y^3)-6*x*exp(-x^2-y^2)*y^2`
`-6*y*exp(-x^2-y^2)*x^2`

3. `-2*(x^3+y^3)*exp(-x^2-y^2)+4*x^2*exp(-x^2-y^2)*(x^3+y^3)`
`+6*y*exp(x-^2-y^2)`

4. `12*x*y^3*exp(-x^2-y^2)*(x^3+y^3)+6*exp(-x^2-y^2)*x`

e) $f_{yy} =$

(7) Circle one:

1. `12*y^3*exp(-x^2-y^2)*(x^3+y^3)+6*exp(-x^2-y^2)*y`
`-12*y^3*exp(-x^2-y^2)`

2. `4*x*exp(-x^2-y^2)*(x^3+y^3)-6*y*exp(-x^2-y^2)`
`-6*x^2*exp(-x^2-y^2)`

3. `-6*y*exp(-x^2-y^2)+12*y^3*exp(-x^2-y^2)-2*(x^3-y^3)*exp(-x^2-y^2)`
`+4*(x^3-y^3)*y^2*exp(-x^2-y^2)`

4. `-2*(x^3+y^3)*exp(-x^2-y^2)+4*y^2*exp(-x^2-y^2)*(x^3+y^3)`
`-12*y^3*exp(-x^2-y^2)+6*y*exp(-x^2-y^2)`

4. Graph the contours (level curves) for this function (use n = 10).

   (8) **Attach your graph to the worksheet.**

5a) The gradient of $f(x, y)$, denoted: $\nabla f(x, y)$ is

(9) Circle one:

1.

$$\big(3x^2e^{-x^2-y^2} - 2x(x^3 - y^3)e^{-x^2-y^2}\big)i + \big(-3y^2e^{-x^2-y^2} - 2y(x^3 - y^3)e^{-x^2-y^2}\big)j$$

2.

$$\big(2ye^{-x^2-y^2}(x^3 + y^3) + 3e^{-x^2-y^2}y^2\big)i - \big(2xe^{-x^2-y^2}(x^3 + y^3) + 3e^{-x^2-y^2}x^2\big)j$$

3.

$$\big(-2(x^3+y^3)e^{-x^2-y^2} + 4x^2e^{-x^2-y^2}(x^3+y^3)\big)i - \big(2(x^3+y^3)e^{-x^2-y^2} + 4y^2e^{-x^2-y^2}(x^3+y^3)\big)j$$

5b) If the contours (or level curves) are far apart, is the $\|\nabla f\|$ large or small? Why?

(10) Circle one:

1. The $\|\nabla f\|$ is small because it is always proportional to the magnitude of $f_{xx}f_{yy} - f_{xy}^2$

2. $\|\nabla f\|$ is large, because a large $\|\nabla f\| \rightarrow f_x$ and f_y are large, implying less contour lines.

3. $\|\nabla f\|$ is small, because a small $\|\nabla f\| \rightarrow f_x$ and f_y are small, implying less contour lines.

4. There is no relationship between the gradient and contour lines.

5c) If you were on the surface at a point (x, y), in what direction would you move to increase your altitude as fast as possible? Why?

(11) Circle one:

1. Opposite to the direction of the gradient, as it points in the direction of greatest ascent.

2. In a direction perpendicular to the direction of the gradient, since it lies in the xy plane.

3. In the direction of the gradient, since it points in the direction of greatest descent.

4. In the direction of the gradient, as it points in the direction of greatest ascent.

6a) At which points (x, y) does f(x,y) have critical points?

(12) Circle one:

1.

$$(0,0),\ \left(0, \frac{\sqrt{6}}{2}\right),\ \left(0, -\frac{\sqrt{6}}{2}\right),\ \left(\frac{\sqrt{6}}{2}, 0\right),\ \left(-\frac{\sqrt{6}}{2}, 0\right)$$

2.

$$(0,0),\ \left(0, \frac{\sqrt{6}}{2}\right),\ \left(0, -\frac{\sqrt{6}}{2}\right),\ \left(\frac{\sqrt{6}}{2}, 0\right),\ \left(-\frac{\sqrt{6}}{2}, 0\right),\ \left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right),\ \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\right)$$

3.

$$(0,0),\ \left(0, \frac{\sqrt{6}}{2}\right),\ \left(0, -\frac{\sqrt{6}}{2}\right),\ \left(\frac{\sqrt{6}}{2}, 0\right),\ \left(-\frac{\sqrt{6}}{2}, 0\right),\ \left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right),\ \left(-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\right)$$

6b) At which points (x, y, z) do the relative extrema occur? (that is, find the matching z value for each critical point):

(13) Circle one:

1.

$$(0,0,0)\ ,\ \left(0, \frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(0, -\frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

$$\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right)\ ,\ \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)$$

2.

$$(0,0,0)\ ,\ \left(0, \frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(0, -\frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)\ ,\ \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

$$\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)\ ,\ \left(-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right)$$

3. not listed

7a. Use the Second Partials Test to determine which critical points yield relative maxima, relative minima or saddle points, if any. What are the MATLAB commands that you used?

- First, what command defines d: (assume that the function $f$and all partial derivatives where defined, i.e., `fx, fy, fxx, fxy and fyy`)
  (14) Answer:

- What command finds the critical numbers and puts these numbers in variables a and b?
  (15) Answer:

- What commands perform the second partials test on the first critical number (a, b)?
  (16) Answer:

7b) At which points (x, y, z) do the **relative maxima** occur?
(17) Circle one:

1.
$$\left(0, -\frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right), \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)$$

2.
$$\left(0, \frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right), \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

3.
$$\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right), \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)$$

4. not listed

7c) At which points (x, y, z) do the **relative minima** occur:
(18) Circle one:

1.
$$\left(0, -\frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right), \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)$$

2.
$$\left(0, \frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right), \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

3.
$$\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right), \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)$$

4. not listed

7d) At which points (x, y, z) do the **saddle points** occur:

(19) Circle one:

1.

$$\left(0, -\frac{\sqrt{6}}{2}, \frac{3\sqrt{6}}{4e^{3/2}}\right), \left(\frac{\sqrt{6}}{2}, 0, \frac{3\sqrt{6}}{4e^{3/2}}\right)$$

2.

$$\left(0, \frac{\sqrt{6}}{2}, -\frac{3\sqrt{6}}{4e^{3/2}}\right), \left(-\frac{\sqrt{6}}{2}, 0, -\frac{3\sqrt{6}}{4e^{3/2}}\right)$$

3.

$$\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{4e^{3/2}}\right), \left(\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{4e^{3/2}}\right)$$

4. not listed

8. Type <u>**hold on**</u> to hold the contour graph and use the **gradient** and **quiver** commands to draw a plot of the gradient vectors. Use the **text** command to label the local extrema and saddle points on this plot. Label any local maximum 'max ', local minimum 'min' and saddle points 'sdl '.
(20) **Attach your graph to the worksheet.**

8a) Around a local minimum, in what directions do the gradient vectors point? Why?

(21) Circle one:

1. The vectors point away from the minimum, because the gradient points in the direction of most rapid ascent.

2. The vectors point in towards the minimum, because the gradient points in the direction of most rapid descent.

3. The vectors point every which way, since there is no relationship between the gradient and extrema.

8b) Around a local maximum, in what directions do the gradient vectors point? Why?

(22) Circle one:

1. The vectors point away from the maximum, because the gradient points in the direction of most rapid ascent.

2. The vectors point in towards the maximum, because the gradient points in the direction of most rapid ascent.

3. The vectors point every which way, since there is no relationship between the gradient and extrema.

8c) Around a saddle point, what happens to the directions of the gradient vectors? Why?

(23) Circle one:

1. The vectors point every which way, since there is no relationship between the gradient and extrema

2. The vectors point away from saddle point, because the gradient points in the direction of most rapid descent.

3. The vectors point both toward and away from the saddle point, as it is both a maximum and a minimum.

MTH233

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Double Integrals

1 New MATLAB Commands

int(int(f,variable,a,b),variable,c,d) is the MATLAB command that computes a double integral. Here **variable** should be replaced by the appropriate symbol, such as x or y, a is the lower bound of the variable and b is the upper bound of the variable. The function f must be in symbolic form, that is $f =$ and no dots should be used. If the bounds of the variables are functions, these should also be given in symbolic form.

Example 1:
To find the volume bounded by $z = x^4 - x^2 + y^3 + y^2$, $z = 0$, $y = 0$, $y = x/2$ and $x = 4$, we integrate

$$\int_0^4 \int_0^{x/2} (x^4 - x^2 + y^3 + y^2) dy dx$$

```
>> syms x y
>> f= x^4-x^2+y^3+y^2
>> int(int(f,y,0,x/2),x,0,4)
```

MATLAB responds with,
ans = 1576/5

Exercise 1:
Use MATLAB to plot the boundary of the region in the xy-plane which is represented in the integrals below. Use MATLAB to compute the double integral. Then reverse the order of the integrals and write the bounds for the reverse order of integration. Use MATLAB to compute the new double integral. You should obtain the same answer either way.

Part A Find the values for a, b, c and d (and use for the following)

$$\int_0^9 \int_{\sqrt{x}}^3 x^2 + y^2 dy dx = \int_a^b \int_c^d x^2 + y^2 dx dy$$

a1.) Generate a graph which shows the region in the xy plane:
(1) **Attach your graph to the worksheet.**

Assume for the following that we have defined:

```
>> syms x y
>> f=x^2+y^2
```

a2.) What is the MATLAB command that integrates

$$\int_0^9 \int_{\sqrt{x}}^3 x^2 + y^2 dydx$$

(2) Circle one:
1. `int(int(f,x,sqrt(x),3),y,0,9)`
2. `int(int(f,y,0,9),x,sqrt(x),3)`
3. `int(int(f,y,3,sqrt(x)),x,9,0)`
4. `int(int(f,y,sqrt(x),3),x,0,9)`

a3.) Now find the bounds, a, b, c and d for the reverse the order of integration.

$$\int_a^b \int_c^d x^2 + y^2 dxdy$$

What MATLAB command evaluates this integral?
(3) Circle one:
1. `int(int(f,y,0,3),x,0,y^2)`
2. `int(int(f,x,0,y^2),y,0,3)`
3. `int(int(f,y,0,y^2),x,0,3)`
4. `int(int(f,x,0,3),y,0,y^2)`

a4.) What answer did MATLAB give?
(4) Answer: ______________________________

Part B Find the values for a, b, c, d, e, f, g and h (and use for the following)

$$\int_0^1 \int_{2-x}^{x+2} x^2+y^2 dydx + \int_1^2 \int_{2-x}^{4-x^2} x^2+y^2 dydx = \int_a^b \int_c^d x^2+y^2 dxdy + \int_e^f \int_g^h x^2+y^2 dxdy$$

b1.) Generate a graph which shows the region in the xy plane:
(5) **Attach your graph to the worksheet.**

b2.) What is the MATLAB command that integrates $\int_0^1 \int_{2-x}^{x+2} x^2 + y^2 dydx + \int_1^2 \int_{2-x}^{4-x^2} x^2 + y^2 dydx$

(6) Circle one:

1. `int(int(f,y,2-x,4-x^2),x,1,2)+int(int(f,y,2-x,x+2),x,0,1)`
2. `int(int(f,y,x-2,4-x^2),x,0,1)+int(int(f,y,x-2,x+2),x,1,2)`
3. `int(int(f,y,2-x,x+2),x,1,2)+int(int(f,y,2-x,4-x^2),x,0,1)`
4. not listed

b3.) What is the MATLAB command that integrates

$$\int_a^b \int_c^d x^2 + y^2 dxdy + \int_e^f \int_g^h x^2 + y^2 dxdy$$

use the bounds you found for a, b, c, d, e, f, g, h

(7) Circle one:

1. `int(int(f,x,y-2,sqrt(4-y)),y,0,2)+int(int(f,x,2-y,sqrt(4-y)),y,2,3)`
2. `int(int(f,y,2-y,sqrt(4-y)),x,0,2)+int(int(f,y,y-2,sqrt(4-y)),x,2,3)`
3. `int(int(f,x,2-y,sqrt(4-y)),y,0,2)+int(int(f,x,y-2,sqrt(4-y)),y,2,3)`
4. not listed

b4.) What answer did MATLAB give?

(8) Answer: ________________________________

Exercise 2:

a.) Use MATLAB to plot the volume bounded by

$$z = 20e^{x^3/8},\ y = 0,\ y = x^2,\ and\ x = 2$$

Use view and axis to get a good orientation that shows both the surface and the region in the xy-plane. Label the graph with a title indicating what view you used. Use `title('view(?)')` to do this.

(9) **Attach your graph to the worksheet.**

b.) Set up the double integral needed to compute the volume under the surface and above the region in the xy-plane, as given in part (a), and find this volume using MATLAB.
What is the MATLAB command you used to compute this integral? Assume `syms x y` and `f=20*exp(x^3/8)` have been defined. *Hint: One order of integration is "non-integrable"! Is it* $x - simple$ *or* $y - simple$*?*

(10) Answer:

c.) What is the numerical value for the volume?
(11) Answer: ______________________________

Exercise 3:

a.) Use MATLAB to plot the solid bounded by

$$z = 54e^{-(x^2+y^2)/4},\ z = 0,\ and\ x^2 + y^2 = 16$$

Use view and axis to get a good orientation that shows both the surface and the region in the xy-plane. Label the graph with a title indicating what view you used. Use `title('view(?)')` to do this. (12) **Attach your graph to the worksheet.**

b.) Set up the double integral needed to compute the volume under the surface and above the region in the xy-plane, as given in part (a), and find this volume using MATLAB.
What is the numerical value for the volume?
(13) Answer: ______________________________